Poznań Reasoning Week

Games and Reasoning | Logic & Cognition | Refutation Symposium

ABSTRACTS

11–15 September 2018, Poznań

About

Poznań Reasoning Week 2018 consists of three conferences, aimed at bringing together experts whose research offers a broad range of perspectives on systematic analyses of reasoning processes and their formal modelling. PRW 2018 is co-organised by the Institute of Psychology, Adam Mickiewicz University and Institute of Philosophy, University of Zielona Góra.
In 2018 we address:

- games in reasoning research (*Games and Reasoning 2018*);
- the interplay of logic and cognition (*Logic and Cognition 2018*);
- refutation systems (*Refutation Symposium 2018*).

Key-notes

Camillo Fiorentini (University of Milano)
Valentin Goranko (Stockholm University)
Gabriele Pulcini (New University of Lisbon)
Hans Tompits (Vienna University of Technology)
Heinrich Wansing (Ruhr-University Bochum)
Keith Stenning (University of Edinburgh)
Gerhard Minnameier (Goethe University Frankfurt)

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Games and Reasoning (GaR 2018)

How not to play the scientific discovery-game

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Keywords: significance testing, discovery, t-test

If the pursuit of goal G1 by playing an actually possible strategy S1 is strictly dominated by another actually possible strategy S2, yet by far most agents in some domain do nevertheless play S1, then it follows—under standard assumptions on human rationality, and for analytical reasons alone—that agents are either ignorant of this dominance relation, or otherwise knowingly deceive (others and perhaps themselves too) regarding the fact that G1 is a merely an alleged or "rhetorical" goal.

This talk shows on formal, meta-methodological grounds that there are strong reasons to believe that the above is the case today in the empirically operating social sciences, with respect to the why and how of making *discoveries* by means of statistical inference methods. Broadly, agents play the scientific discovery game in ways that cannot plausibly foster the development of theoretical knowledge.

To develop the empirical social sciences further, after all, we require well-supported theoretical constructs. To justify the effort of developing such a construct, researchers need to supply theoreticians with empirical data that support non-random effects whose replication probability is sufficiently high. For it is only such effects that she *should* seek to subsume under a theoretical construct in the first place.

To establish such effects statistically, researchers (rightly) rely on a t-test, which originates with Neyman-Pearson test-theory (NPTT) [2]. By contrast, Fisher's [1] classic approach to statistical inference did only employ the α-error (aka p-value), using it to narrowly evaluate the probability of data under the H_0, $p(D, H_0)$. Since this disregards the H_1, it thus is for trivial reasons that his approach cannot offer a reproducibility measure.

Improving Fisher's approach, Neyman-Pearson test-theory (NPTT) [2] lets α- and β-error jointly evaluate whether data are more compatible with the H_1 than with the H_0. Together, both errors measure whether data are *stable*, that is, non-random *and* replicable. Whereas instable data are prone to mislead, stable data alone can meaningfully sustain an induction over them.

The full set of relevant NPTT-parameters is as follows:

(i) the α-error: the probability of falsely judging that empirical data deviate from the H_0 (false positives);

(ii) the β-error: the probability of falsely judging that data deviate from the H_1 (false negatives);

(iii) the $1 - \beta$-error (aka 'test-power'; derived from (ii)): the probability that a replication-attempt duplicates an original data-signature, here hypothetically granting the H_1 as true;

(iv) the effect-size, d: the theoretically expected influence from independent onto dependent variables;

(v) the minimum sample-size, N_{min} (derived from α, β, d), required to register a statistically significant non-random effect given stable data.

The relevant effect-size measure for a t-test is the difference (d) between two means divided by the standard deviation of data, $d = (m_1 - m_2/s)$, where $0 \leq |d| \leq \infty$. (The praxis of representing *inverse effects* on the negative arm of the number line is a mere convention.) Generally, d = 0.01 counts as a *very small* effect; d = 0.20 is *small*; d = 0.50 *medium*; d = 0.80 *large*; d = 1.20 is *very large*; and some call d = 2.0 *huge*. While other quantitative measures for the magnitude of a phenomenon include the correlation measure r and Hedges' g, for instance, d probably is the most widely used measure. If the base-rates in experimental- and control-group are the same, moreover, we can safely transform d into either r or g.

Since NPTT has three degrees of freedom, fixing any three of α, β, d, and N_{min} lets the fourth parameter fall out. This interdependence recommends NPTT as a planning tool, for it quantifies N_{min} *before* data collection starts. Through adopting specific α- and β-errors and by fixing a minimum d via theoretical reasoning, we thus deduce N_{min}. Samples smaller than N_{min}, after all, will not do, for to sustain a theoretical construct requires stable data. *Ceteris paribus*, then, this fully determines the *necessary* cost of data-collection. (Indeed, N_{min} is the one material resource we *can* determine. By contrast, α, β are normative parameters, while d is either an empirically obtained or a theoretically predicted parameter.)

But many employ one or both of two strategies that reduce the cost of data-collection. Both strategies lower N_{min}. The first strategy increases the β-error; while the second treats the control-group as a constant, thereby collapsing a two-sample t-test into its one-sample variant. A two-sample t-test of d = 0.50 with N_{min} = 176 under $\alpha = \beta$ = 0.05, for instance, becomes a one-sample t-test with N_{min} = 27 under α = 0.05, β = 0.20.

Not only does this decrease the replication-probability of data by a factor of four. Such data also cease to corroborate empirical hypotheses. The ubiquity of both strategies, we submit, therefore is a partial cause of the ongoing confidence-crisis in the social sciences as elsewhere. Since research groups may pool their samples until they reach N_{min} jointly, moreover, actual resource restrictions sufficiently justify neither strategy.

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Process of reasoning as a game: schema and conditions

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Keywords: game-reasoning, question-answer component, modeling reasoning

Any text (scientific, journalistic, pictorial or literary) always tacitly contains an active question-answer component. In other words, we enter into a dialogue between the author (text) and the reader/viewer.

Each text is original and therefore offers its own unique set of sings that the reader/viewer should decode. The reader, guessing the author's idea, always argues, answering himself to the questions *why*? *how*? *what*?. However, it should be noted that the process of reasoning itself takes the form of a game. The game can be understood as meaningful activity of a person, which is aimed at modeling some specific activity, and the motive of the game will not be its result, but its process. For example, we offer a child a game. What does the child do? He builds possible worlds, both his behaviour (and the behaviour of other participants), and his speech. The child comes up with a text that accompanies his actions, or vice versa. Actions may not be (for example, if the child is sick or cannot move), but the text, verbal or non-verbal, is always there.

What influences the text-reader/viewer dialogue and how is the reasoning game built when working with text? First, any text is closed. It is closed by the position of the author, who offers his concept. In this respect, the scientific text is more complex than any other. For example, an artist can change the styles, a writer can write a children's tale, or can write a historical novel. Secondly, the text offers its own conceptual apparatus, or its own sign system. Thirdly, the text is an integral coherent chain of reasoning, a system of play. However, for example, in a picturesque text such a chain of reasoning can be non-verbal. Or, for example, in a literary or pictorial text the game is present, but less is felt just as a reasoning game, because a literary work often gives ready images, offering morality, a ready-made look at something also offers a picturesque text. In the scientific text, we follow the author in his reasoning; we try to understand why he is building his conceptual position in his own way.

If such a game-reasoning is presented in the form of a scheme: speaker (text) and listener/reader, then such a scheme does not indicate anything new in the process of reasoning, but only repeats the well-known form of dialogue as the perceptions of any text. The main thing for us to find, the world of the text (the author) and the reader's world will be connected. The defining characteristic for these worlds will be the sign system. It determines the background in which each of these worlds is located. These worlds will be connected with each other by reasoning constructed in the form of a question-answer.

Agreeing with Hintikka, who argued that building reasoning, we create a certain model of a certain world.

Why do we ask a question? We ask a question because we have two models of the world that do not coincide. It's one thing when we ask a question to a person who is together with us in the same sign system and quite another thing, when the sign systems are determined by epochs, views, reading of a particular literature, opinions, etc.

We ask a question, because we do not know or do not understand how the other world that is offered to us is possible. We play with the child here and now. We are in the same sign system.

Let us consider such a text as a riddle. The riddle can be defined as a text whose denote is an object, in the text itself it is not explicitly named. This allows us to assert that the riddle does not pretend to unambiguous understanding. As an independent text, the riddle introduces us to the field of sign and meaning. However, the basic position, when considering the inner structure of the riddle, should be the birth of a new meaning for not always, at first glance, comparable objects. The riddle offers a text on the basis of which the guesser proposes another text by comparing objects, and thus posing the question: what can it be? How can this be, etc.? For example, consider two riddles about the rainbow: (1) I am purple, yellow, red and green/ the King cannot reach me and neither can the Queen/ I share my colours after the rain/ and only when the sun comes out again; (2) I am multi-coloured/ I appear after a storm/ People always point at me/ Everyone takes my picture/ Legend says there is gold at the bottom of me.

The first riddle is very easy to guess, because it is difficult to find any other object with the same characteristics. And the second riddle suggests the construction of several possible worlds for its solution, we reason, analyzing the subject, which is in question in the riddle.

So, the perception of the text occurs in several planes. First, it is a background in which a text or sign system was created that determines the conceptual level of the text, its language background. Secondly, this is the level of the reader/viewer's entry into the character system of the text. The level of entry into the sign system or, in other words, the index of understanding the sign will be determined by understanding the content of the text by building a chain of reasoning that will be a variant of the interpretation of the text or reasoning-game, and, thirdly, a plane of truth, taking into account previous aspects.

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On a simple model of a simple game: ‘Guess Who?’ and Inferential Erotetic Logic

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Keywords: ‘Guess Who?’, Inferential Erotetic Logic, polar questions, situational semantics

In this talk we introduce a model for playing ‘Guess Who?’ game, developed within the framework of Inferential Erotetic Logic, and its refinement elaborated on in terms of situational semantics.

‘Guess Who?’ is a two-player character guessing game. Its objectives may be concisely characterised in a way for which a quotation from the Board Game Capital game manual (`http://www.boardgamecapital.com`) is a good example:

> ‘Guess Who?’ is a classic two player game where opponents attempt to guess which character out of 24 possibilities their opponent has picked. Questions such as “Are you wearing glasses?” can help narrow down your choices. But choose your questions wisely and don’t let your opponent name your character first!

Our main aim is to give the precise meaning to the concept of a “wise choice” of a question. To this end we shall introduce the concepts of a discriminatory power of a polar question and of a history of a question. We shall introduce a formal model for the game employing the concept of erotetic implication [3] as the underlying yardstick for normatively correct interrogative moves in the game. Further on, we shall refine this model in order to account for dynamics of information processing stemming from asking and answering auxiliary questions. Formal basis for this refinement is offered by situational semantics [2; 4]. Finally, we shall reformulate some known strategies of playing ‘Guess Who?’ [1] in terms of our models.

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Modeling Bottom-up Cognition in the Card Game SET

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Keywords: visual search, SET card game, Decision-making tasks

Decision-making tasks generally involve both bottom-up and top-down processes, but the relation between the two is not clear. Bottom-up processes are perceptual, and rely on visual features such as color, shape, or size; while top-down processes are conceptual and goal-driven. Lets consider two scenarios: one in which I am searching for my jacket on a coat rack in a restaurant, and the other, I am in a store searching for a jacket to purchase. In the former case, I rely on my top-down process, as I have a certain goal in mind and have a specific concept (my jacket) that is guiding my visual search. In the latter case, I do not have this concept in mind, I am relying on visual stimuli to attract my attention. That is, as I am scanning my environment, I am relying on salient features to attract my attention. But in the store, is my decision about which jacket I choose to purchase solely based on bottom-up elements within the visual scene? Or does the saliency of the object in an explorative scanpath rely on bottom-up and top-down cognition? If a jacket attracts my attention because it has bright colors, then the elements are bottom-up. But if am searching the store for a certain type of jacket, then my search is guided by a conceptual goal.

At first glance it seems that in the latter scenario my search is random, and my decision for which jacket I purchase is heavily influenced on the perceptual properties of the jacket. While my decision is indeed heavily based on the perceptual properties, it is not always the case that my search is random. Even if we assume that I have no previous knowledge of the store or the jackets in consideration, I am still guided by a conceptual goal, viz. the type of jacket I have in mind. In order to investigate the interaction between bottom-up perception and top-down planning within a visual search, I use the card game SET. SET, The Game of Visual Perception provides a fruitful environment for modeling decision-making tasks, especially when dealing with the interplay between bottom-up and top-down cognitive processing. More specifically, I shed to light a discrepancy between the objective probabilistic occurences of types of SETs and the players subjective preference to them. I then suggest a strategy that models the interplay between the players bottom-up preference and top-down planning.

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Natural Deduction Method for Solving CL-based Puzzles

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Keywords: Games and Reasoning, combinatory logic, combinator, reduction, natural deduction, puzzle solving

Combinatory logic (CL, for short) is one of the two formal systems focused on the notion of function introduced in 1920s (the other one being lambda calculus). The main aim of it was to eliminate bound variables. Additionally, CL is closely related to lambda calculus. CL is mainly concerned with examining ways in which simpler functions can be combined into other, more complex ones. Such functions are called combinators. CL has been developed in 1926 by Moses Schöfinkel; at the same time Haskell Curry began working on simplifying the process of substitution [2]. Additionally, he introduced a formal representation of building terms from variables and constants. As a result, language of CL is very concise—it consists of variables and constants which can be combined to form terms and formulas. CL has been designed to solve the problem of a substitution, which is connected to the bound variables. For example, in classical first-order logic variable occurrences and the substitution is defined by means of certain restrictions. On the other hand, the process of substitution in CL is a simple and direct one—the most substantial advantage of CL is the lack of bound variables [8]. Due to it CL has found its application in many different fields, mostly in computer science and mathematics. An interesting approach to CL comes from Raymond Smullyan, who proposed a number of puzzles based on CL [6]. These puzzles show certain properties of combinators and rules that can be applied to them. Smullyan presents these through descriptive and simple manner. A number of well-known combinators and the way they are combined is presented by way of analogies. Functions are represented by birds which are characterized by different calls and responses. Also, as Smullyan's way of presenting the puzzles provides an insight into a structure of an algorithm that could be formalized, many formal methods adapted to solving the puzzles can be designed. So far, methods based on analytic tableaux [1] and graph-based approach [5] have been introduced along with Prolog implementations [3; 4]. I propose a simple and concise method of solving Smullyan's puzzles which is based on the natural deduction. The choice of natural deduction method comes from the correspondence (which has been noticed by Curry) between intuitionistic formulae and type expressions, as well as between proofs and lambda terms [7]. Combinatory logic in many ways is similar to lambda calculus. Combinators can be translated to lambda terms (and vice-versa), therefore many of the proposed methods can be strikingly similar. The process of normalization of natural deduction can be interpreted as the one of program evaluation (looking for a beta normal form of a lambda term). The proposed normative way of solving the puzzles is based on the selected deduction rules for CL—that is, rules utilizing the properties of transitivity and commutativity. Other rules are concerned with the substitution. Proofs for a given puzzle are thus represented by trees governed by introduced rules. We build trees in a top-down manner, by starting with leaves labelled with preassumed conditions (conditions-axioms) to the root labelled with a combinator we are supposed to derive. As the level of complexity of the puzzles is increasing, the set of rules sufficient for solving simpler puzzles can be expanded. Also, adding derived combinators to the list of conditions-axioms allows to prevent unnecessary expansion of the tree. Proof trees for a given puzzle are obtained from Smullyan's solutions expressed in natural language. Aside from showing natural deduction method, I will give examples of other methods for solving Smullyan's CL-based puzzles, in which bound variables are included. I will show some limitations of them and problems that arise when bound variables are present.

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The influence of real-time feedback on quality of collected data in GWAP

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Keywords: crowdsourcing, games with a purpose, quality control

Crowdsourcing is a popular method of outsourcing tasks to non-experts used in order to acquire large amounts of data or to solve a problem that can not be solved automatically. The most frequently mentioned benefits of applying this approach are related to the fact that it is often cheaper and faster than in the case of cooperation with experts in a given field.

The profitability of using crowdsourcing to solve the problem can be considered in three dimensions: costs (how much the task will cost), latency (time of waiting for the effect), quality (quality of the obtained data) [3].

One of the most interesting solutions that are used to make crowdsourcing process even cheaper and faster is designing it as a games with a purpose (see examples: Phrase Detective[1], Wordrobe[2], Foldit[3]). Such games are primarily intended to reduce project costs because a well-designed and engaging game has a chance to reach a large audience. In practice, however, the authors of GWAPs often decide to use a financial incentive, because it is a strong motivator for players [1]. Increasing the number of annotators and the appropriate design of the annotation process may positively affect the latency [3]. However, using the GWAPs does not help to solve the problem of the quality of collected data (which is also faced with traditional crowdsourcing).

Crowdsourcing researchers distinguish the division of quality control approaches into two groups: procedure-driven and data-driven [2]. Data-driven methods include mathematical models that are used to analyze responses or users' behavior in order to obtain a high-quality answers (e.g. Inter-annotator agreement, expectation maximization models).

Data-driven quality control methods are the object of many studies in the field of crowdsourcing and have been proven to have a positive impact on data quality [4]. Due to the fact

[1]`http://anawiki.essex.ac.uk/phrasedetectives/`
[2]`http://wordrobe.housing.rug.nl/Wordrobe/public/HomePage.aspx`, access date: 28 October 2016.
[3]`https://fold.it/portal/`

that these methods depend on the obtained data and not the system itself, they are often also successfully used in GWAPs.

Procedure-driven quality control methods are associated with the annotation process (such as the way of decomposing the task, working in cycles, the design of annotation interface). These methods are less developed and tested compared to data-driven. Because of their design, GWAPs can be interesting subject of research in this area, because they have a more complex interface compared to traditional data acquisition tools, have long loops of the game and decompose tasks in various ways. A particularly neglected procedure-driven quality control method is providing real time feedback to the annotators. The right feedback can affect the quality of data during annotation—increase or decrease it.

In the speech, we will talk about the authorial platformed called SpaceTag, created for cross-sectional research on quality control methods in crowdsourcing used for the purpose of gathering linguistic data with particular emphasis on the influence of real-time feedback during the annotation process. SpaceTag is in development stage, it contains game used for collecting text annotations and allows automatic A/B testing that are used to analyse different approaches. We will also talk about the problem of feedback, the first pilot studies carried out using the SpaceTag platform and the conclusions we achieved regarding the data quality.

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Part III

Logic and Cognition (L&C2018)

Cognitive Principles and Differences in Human Syllogistic Reasoning

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Psychological experiments have confirmed that humans do not reason according to Classical Logic (cf. [2; 14]). Therefore logic-based approaches in general might not be suitable for understanding and formalizing episodes of human reasoning. We are convinced that logic can help us better understand why humans draw certain conclusions, but claim that Classical Logic is not adequate for this purpose. According to [13], theories that aim at modeling human reasoning need to be assessed by their cognitive adequacy, i.e. they need to be evaluated based on how humans actually reason.

During the last decades, various cognitive theories have been proposed (cf. [3; 4; 7; 10; 11]). However, the results of the meta-study in [8] based on six experiments on syllogistic reasoning are disappointing: Humans do not only systematically deviate from the assertions in Classical Logic, but from any other proposed cognitive theory. Syllogisms originate from Aristotle [1] and they are especially interesting in the field of Psychology, because on one hand they are easy to understand and on the other hand they are complex enough to require actual reasoning. A syllogism consists of two premises and one conclusion. The assertions are statements about properties over terms and can have one of the classical quantifiers, i.e. *For all a are b, Some a are b, Some a are not b or No a is b*, which are called moods. The pair of syllogistic premises can have four different orders. Based on these classical moods and orders, 64 different pairs of syllogistic premises can be constructed. A conclusion then consists of an assertion about two properties over terms where each of these properties occurs in only one of the premises, together with a third middle property that does not appear in the conclusion but appears in both premises. Consider the following pair of syllogistic premises:

Some artists are bakers. *Some bakers are chemists.*

The task is then: Given these two premises, what conclusions on the relation between *artists* and *chemists* are valid? Classical logically, *no valid conclusion* follows from these premises. However, according to [8], the majority of participants in experimental studies concluded that *some artists are chemists* (62%) and *no valid conclusion* (34%) follow.

A major drawback of the suggested cognitive theories so far is that only the participants' aggregated data has been considered, i.e. these theories do not account for individual differences.

Consider again the above example: The majority's conclusions exclude each other, participants who answered *no valid conclusion* are not likely to be the same ones who answered *some chemists are artists*. A cognitive theory that claims to be adequate should account for these differences. We need a theory that identifies the clusters of reasoners and models how they come to different conclusions given the same information.

[9] proposed an approach to model the individual reasoning patterns of the participants. They identified three different reasoning clusters: The intuitive cluster, the intermediate cluster and the deliberative cluster. Another preliminary clustering approach has been suggested in [5], which puts forward the hypothesis that the clusters can be expressed through cognitive principles. Cognitive principles are (not necessarily valid) assumptions made by humans while reasoning, such as licenses for inferences [12] or existential import [6]. We re-assess this approach and investigate whether the proposed clusters adequately fit the non-aggregated dataset provided by [9].

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Inferential Knowledge and Knowledge Representation

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What kind of knowledge does one get from an inference? Such a question might appear as awkward, since the answer seems so obvious: From an inference, one gets knowledge *simpliciter*. In the field of artificial intelligence (AI), for instance, expert systems exploit knowledge bases by inferential means in order to acquire knowledge. Inference engines in expert systems do not alter the epistemic properties of epistemic items that are parts of an inference, i.e., the inferred knowledge item is of the same type as the knowledge in the premises [2]. In the field of epistemic logic, knowledge of a formula ϕ is defined as the truth of ϕ in all accessible states (worlds), and the knowledge operator is interpreted in exactly the same way at every state [4]. One common feature of these two perspectives on knowledge is that the concept of knowledge is conceived as being *univocal* and as being closed under material implication. In other words, if one knows that ϕ and one knows that $\phi \supset \psi$, then one knows *in the same sense* that ψ.

A univocal concept of knowledge is clearly adequate when an agent is drawing conclusions from a single knowledge base, or even many, if all the declarative knowledge involved is of the same type. But what if different types of knowledge (concepts of knowledge) are intermingled? For instance, if one has *perceptual* knowledge that ϕ and *logical* knowledge that $\phi \supset \psi$, then what kind of knowledge (if any) does one have that ψ? I defend the idea that, when reasoning about knowledge, inferences are sensitive to the variety of knowledge types, and that in a knowledge representation where there are no distinctions between knowledge types (i.e. when knowledge is strictly conceived as a univocal concept), inferences may generate epistemic equivocity, in the sense that epistemic operators may become equivocal.

In my view, the Muddy Children Problem (MCP) provides a good conceptual setting for putting into light and for analyzing the difficulty posed by epistemic equivocity for inferential knowledge. In AI and in logic, the MCP serves the purpose of illustrating common knowledge and its properties [1; 3; 5]. Common knowledge is a group knowledge, as universal knowledge and distributed knowledge (or tacit knowledge), and it is the basis of the MCP-theorem. But, there is more to the MCP. In the MCP, two types of knowledge are interacting, namely perceptual knowledge and logical knowledge, and by emphasizing their interplay one can bring to the foreground an important epistemological difficulty. It is possible to show that, in the absence of a formal distinction between the different types of knowledge involved, the MCP implies some sort of contradiction, i.e., for some proposition ϕ, an agent knows that ϕ while she does not know that ϕ. In this particular case, some children know that they are muddy while not knowing that they are muddy. I claim that (1) such a contradiction stems from some epistemic equivocity which is embedded in the knowledge representation, and that (2) by making knowledge types explicit in the representation this contradiction can be avoided.

In the first part of the talk, I will revisit the MCP. I will show that this problem puts also into light some sort of epistemic equivocity between concepts of knowledge, and consequently that the problem calls for some logical refinements with respect to the representation of the types of knowledge involved in an inference. In the second part, I will address this issue from a model-theoretic point of view, and I will develop a fragment of epistemic logic capable of providing a solution to the problem of epistemic equivocity.

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The logic of abduction, deduction, and induction, and a taxonomy of inferential reasoning

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The Peircean concept of "abduction" has been differentiated into different forms and made fruitful in a variety of contexts. However, the very notion of abduction still seems to be in need of clarification. In order to achieve this, I will take very seriously Peirce's claims that (1) there are only three kinds of reasoning, i.e. abduction, deduction, and induction, and that (2) these are mutually distinct. First, I will try to explicate the fundamental features of the three inferences, as I see them. After this, I propose several extensions: One is to take up Peirce's idea of "theorematic deduction", interpret it in terms of inverse deduction and transfer this to conceptualize inverse abduction and inverse induction. Another one is to differentiate cognitive levels (from perception to high-level theorizing), and finally I shall also consider different domains of reasoning (explanatory and non-explanatory).

Human Reasoning, Computational Logic, and Ethical Decision Making

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Computers are getting more and more intelligent, which has as a consequence that they gain independence on taking choices and on interacting with their environment. They are already part of our society, and will have even more impact in our every day life in the future. A typical example are the improvements made in the development of self-driving cars. This technological progress is unavoidable and, therefore, ethical decision making is an emergent topic in this field.

Ethics is a branch of Philosophy that deals with the question of what is right and what is wrong behaviour. In this area, the construction of thought experiments are a common way to illustrate the idea behind a theory. In particular for the case on ethical theories, the trolley problem [1] as well as its variants are helpful to demonstrate ethical dilemmas.

Luís Moniz Pereira and Ari Saptawijaya present a computational logic approach towards programming machine ethics in [3]. They discuss the trolley problem and its variants, and claim that logic programming based techniques are appropriate to formalize these problems. For each problem a unique logic program that represents the scenario is provided, for which a query can be answered on which action is morally permissible. Two drawbacks of the approach are that it does not provide a general method to account for ethical dilemmas in other scenarios and that it is not integrated into a cognitive theory about human reasoning.

In this paper we will consider the weak completion semantics [4] for modeling human reasoning tasks. Under this semantics, each weakly completed logic program $\mathcal{P}$ admits a least model. In all human reasoning tasks which we have considered so far, this least model can be computed as the least fixed point of a semantic operator $\Phi_{\mathcal{P}}$, which was first introduced by Keith Stenning and Michiel van Lambalgen[5]. Then, reasoning is performed with respect to this least model.

In order to model the trolley problem and, in particular, to model actions with direct and indirect effects we propose to integrate the fluent calculus [2] into the weak completion semantics. Within the fluent calculus, a state of affairs is represented by a multiset of fluents. States are changed by the execution of actions. Actions are specified by their preconditions and their direct effects. But actions may have also indirect effects, which can be computed by ramification.

In the fluent calculus multisets and multiset rewritings are specified with the help of an appropriate equational theory. In the full paper, we extend the semantic operator $\Phi_{\mathcal{P}}$ to $\Phi_{\mathcal{P}}^{E}$ such that it can handle equational theories. Moreover, we provide logic specifications for the trolley problem and its variants such that the least fixed point of the extended semantic operator allows to reason about morally permissible actions.

As an example consider the trolley problem: The driver of a runaway tram can steer into one of two tracks (the direct effects) and, thereafter, can only watch while the tram is hitting people on these tracks (the indirect effects). In the initial state, the trolley is on track 0 ($tt(0)$), track 0 is clear ($cl(0)$), track 0 is connected to tracks 1 and 2 via a switch which is initially in position 1 connecting track 0 to track 1 ($sw(1)$), on track 1 there are five humans ($ht(5,1)$), while on track 2 there is one human ($ht(1,2)$). In the fluent calculus the initial state is represented by the following term:

$$tt(0) \circ cl(0) \circ sw(1) \circ ht(5,1) \circ ht(1,2) \tag{1}$$

where $\circ$ is a binary function symbol written infix which is associative, commutative and admits a unit element.

The driver has the choice of executing two actions: he can either do nothing or he can change the switch into position 2 such that the tram is running into track 2 instead of track 1. In the fluent calculus this is represented by a ternary predicate causes with informal meaning that its first argument—the current state—is turned into the third argument—the next state—when the second argument—an action—is executed:

$$causes((1), donothing, (1)) \leftarrow \top$$
$$causes((1), change, tt(0) \circ cl(0) \circ sw(2) \circ ht(5,1) \circ ht(1,2)) \leftarrow \top$$

But there are also indirect effects. These are represented as actions with the help of a ternary predicate *action*. Its arguments are the preconditions, the name, and the direct effects of an action:

$$action(tt(0) \circ cl(0) \circ sw(1), downhill, tt(1) \circ cl(0) \circ sw(1)) \leftarrow \top$$
$$action(tt(0) \circ cl(0) \circ sw(2), downhill, tt(2) \circ cl(0) \circ sw(2)) \leftarrow \top$$

$$action(tt(1) \circ ht(5,1), kill, tt(1) \circ d(5)) \leftarrow \top$$
$$action(tt(2) \circ ht(1,2), kill, tt(2) \circ d(1)) \leftarrow \top$$

The indirect effects are computed as ramifications if nothing abnormal is known:

$$causes(X, A, E \circ Z) \leftarrow action(P, A', E) \wedge causes(X, A, P \circ Z) \wedge \neg ab$$

Nothing abnormal is known:

$$ab \leftarrow \bot$$

Applying the $\Phi_{\mathcal{P}}^{E}$ operator to the program developed above yields a least fixed point after three iterations mapping the atoms

$$causes((1), donothing, tt(1) \circ cl(0) \circ sw(1) \circ d(5) \circ ht(1,2)) \tag{2}$$
$$causes((1), change, tt(2) \circ cl(0) \circ sw(2) \circ ht(5,1) \circ d(1)) \tag{3}$$

to true. Now, the driver can reason that changing the switch causes less humans to die than doing nothing.

Moreover, the driver can also consider the case where there is no human on track 2, which is represented by the following alternative initial state:

$$tt(0) \circ cl(0) \circ sw(1) \circ ht(5,1) \circ cl(2) \tag{4}$$

Adding

$$causes((4), change, tt(0) \circ cl(0) \circ sw(2) \circ ht(5,1) \circ cl(2))$$

to the program and applying $\Phi^E_{\mathcal{P}}$ again yields a least fixed point where in addition to (2) and (3) the atom

$$causes((4), change, tt(2) \circ cl(0) \circ sw(2) \circ ht(5,1) \circ cl(2)) \tag{5}$$

is mapped to true. Comparing (5) with (3), the driver can conclude that the death of the human on track 2 was not intended and, consequently, the execution of the *change* action is permissible. As shown in the full paper, the knowledge necessary for this line of reasoning can also be encoded as a logic program such that $\Phi^E_{\mathcal{P}}$ computes the permissibility of the *change* action.

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Distinguishing argument and explanation with description logic

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Keywords: argument and explanation, reasoning in description logic

Argument and explanation are considered distinct and equally fundamental [2], with a complementary relationship [3]. Even if interleaving argument and explanation is common practice in daily communication, the task of extending argumentation theory with the concept of explanation is still challinging [4]. Preliminary ideas of this study have been presented at the CMNA workshop.

The fusion of argument and explanation is best shown by the fact that humans tend to make decisions both on knowledge and understanding [5]. For instance, in judicial cases, circumstantial evidence needs to be complemented by a motive explaining the crime, but the explanation itself is not enough without plausible evidence [3]. In both situations the pleading is considered incom- plete if either argumentation or explanation is missing. Thus, the interaction between argument and explanation has been recognized as the basic mechanism for augmenting an agent's knowledge and understanding [1], known as the *argument-explanation pattern*.

The role of argument is to establish knowledge, while the role of explanation is to facilitate understanding [3]. Thus, to make an instrumental distinction between argument and explanation, one has to distinguish between knowledge and understanding. We consider the following distinctive features of argument and explanation: (i) *Starting condition*: explanation starts with non-understanding, argumentation starts with a conflict; (ii) *Role symmetry*: In explanation the roles are usually asymmetric: the explainer is assumed to have more understanding and wants to transfer it to the explainee, while in argumentation, both parties start the debate from equal positions; (iii) *Linguistic indicator*: In explanation one party supplies information. There is a linguistic indicator which requests that information; Because in argumentation all parties supply information, no indicator of demanding the information is required; (iv) *Acceptance*: An argument is accepted or not, while an explanation may have levels of acceptance.

Arguments rely on evidence, while explanations on causes (Fig. 1). Moreover, evidence and cause are not disjoint: the same sentence can be interpreted as evidence in one reason and as cause in another reason. Consider the following statements: *John must love speed.* $\ulcorner 1 \urcorner$, *He drives with high speed all the time*. $\ulcorner 2 \urcorner$, *That's why he got so many fines*. $\ulcorner 3 \urcorner$. One interpretation is that statement $\ulcorner 2 \urcorner$ represents the support for statement $\ulcorner 1 \urcorner$. Statement $\ulcorner 3 \urcorner$ also acts as an explanation for $\ulcorner 3 \urcorner$, as suggested by the textual indicator "*That's why*". Fig. 2 illustrates the fomalisation in description logic of these two reasons. Assume that the interpretation function

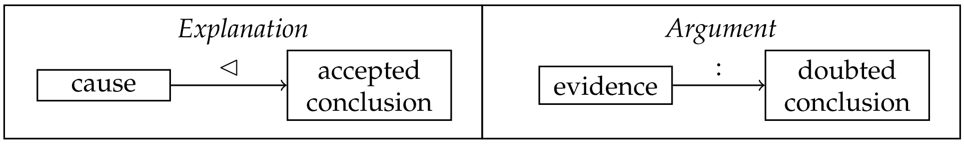

Figure 1: Distinguishing argument from explanation.

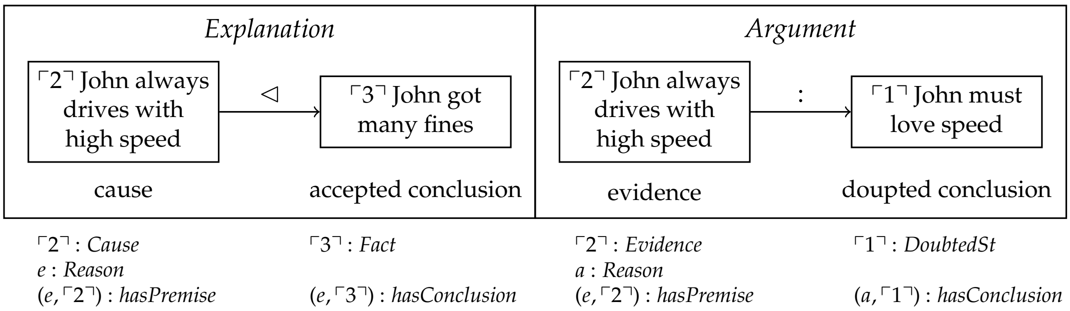

Figure 2: Reasoning in Description Logics. The same statement $\ulcorner 2 \urcorner$ acts as a cause for the accepted statement $\ulcorner 3 \urcorner$ and as evidence for doubted statement $\ulcorner 1 \urcorner$. The agent with this interpretation function treats e as an explanation and a as an argument.

$\mathcal{I}$ of the hearing agent h asserts statement $\ulcorner 2 \urcorner$ as an instance of the concept *Cause* and $\ulcorner 3 \urcorner$ as a *Fact*. Hence, reason e is classified as an explanation. Assume that assertion box of agent h contains also the assertion ($\ulcorner 1' \urcorner$, $\ulcorner 1 \urcorner$): *attacks*. Hence, agent h classifies the statement $\ulcorner 1 \urcorner$ as doubted. Adding that $\ulcorner 2 \urcorner$ is interpreted as evidence, agent h classifies the reason a as an argument.

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The sorites paradox in mathematical fuzzy logic

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Given a vague predicate F, a sorites series is a finite sequence of objects $\langle x_0, x_1, \ldots, x_n \rangle$ such that:

- F definitely applies to x_0,
- F definitely does not apply to x_n,
- for each $i < n$, the objects x_i and x_{i+1} are extremely similar in all respects relevant to the application of F.

Such a series can be translated into the following problematic argument: "x_0 is F; for each $i < n$, if x_i is F, then so is x_{i+1}; therefore x_n is F", which gives a logical paradox (the *sorites paradox*) because it has the form a valid argument whose first premise is clearly true, also the second premise seems true, and yet its conclusion is clearly false.

This paradox has been intensively discussed in the literature by several competing theories of vagueness (see e.g. [1; 2; 5; 6; 8; 10–14]). An important task often mentioned is to explain why ordinary speakers react to it in the way they do:

- They go along with the premisses and the reasoning, at least initially. They do not immediately reject some premise, nor claim that there is a fallacy.
- They do not accept the conclusion. When they see where the argument goes, they baulk. Yet, they are generally unable to say exactly what went wrong. The premisses still seem attractive and the reasoning still seems reasonable—but they certainly do *not* have the considered intuition that the argument is sound (i.e. logically valid, with fully true premisses, and hence a fully true conclusion).

The standard explanation (see e.g. [7]) of this phenomenon using fuzzy logic goes back to Goguen [3]: the premisses of the sorites are all at least *very nearlty* true—so for ordinary purposes we accept them. However, they are not all fully true. So the argument is (valid but) unsound. Formally, this explanation is based on a specific evaluation of formulas in the sorites series assigning to each successive instance a slightly lower truth value form the interval $[0, 1]$ (starting with 1 and ending with 0) and the fact that, using the Łukasiewicz implication, implicative premisses of the argument have a truth a value only slightly smaller than 1.

Hájek and Novák [4], benefiting from a later and more developed stage of mathematical fuzzy logic, presented an alternative formalization of the sorites paradox aimed at emancipating it from ad hoc evaluations in the Łukasiewicz semantics while, allegedly, preserving the main idea of Goguen's solution. Namely, they used the codification of Peano arithmetic inside fuzzy logic expanded with a unary predicate *Fe* (intended to be interpreted as the vague property of *feasible* number, i.e. small), and a unary propositional connective *At* (with the intended meaning of *almost true*), obeying the following axioms:

$(at1)$ $\varphi \to At(\varphi)$

$(at2)$ $(\varphi \to \psi) \to (At(\varphi) \to At(\psi))$

Denoting, as usual, the numerals as $\overline{n}$, the sorites is translated as:

$$\begin{array}{c} Fe(\overline{0}) \\ Fe(\overline{0}) \to Fe(\overline{1}) \\ Fe(\overline{1}) \to Fe(\overline{2}) \\ \vdots \\ \hline Fe(\overline{N}) \end{array}$$

where N is a sufficiently big natural number to produce the paradox (a valid argument where $Fe(\overline{0})$ is clearly true, the implicative premisses are also apparently true, and $Fe(\overline{N})$ is clearly false).

Their new explanation of the sorites is based on the following alternative argument:

$$\begin{array}{c} Fe(\overline{0}) \\ Fe(\overline{0}) \to Fe(\overline{1}) \\ Fe(\overline{1}) \to Fe(\overline{2}) \\ \vdots \\ \hline At^N Fe(\overline{N}) \end{array}$$

where At^N means the consecutive application of the unary connective N times. While the first premise remains the same, the others have been modified to account for the fact that at each step of the series there is indeed some small change in the truth of the predicate (if n is a feasible number, then it is almost true that $n + 1$ is feasible). An easy proof in the axiomatic system shows that the argument is indeed valid, but the modification of the conclusion (not anymore to be taken as obviously false, but actually perfectly acceptable as true) has dissolved the paradox.

The aim of our talk is twofold:

1. We want to discuss an explanation of the sorites paradox based on this formalization by Hájek and Novák. We need to explain why a speaker would go along with the sorites premise $Fe(\overline{n}) \to Fe(\overline{n+1})$, given that he takes $Fe(\overline{n}) \to At(Fe(\overline{n+1}))$ to be fully true. We will do so by appealing to the Ramsey test [9]: the conditional $\alpha \to \beta$ is acceptable to the extent that β seems acceptable after you add α hypothetically to your stock of beliefs

(and then make whatever minimal adjustments are necessary to maintain consistency; NB adding α hypothetically to your stock of beliefs means supposing α to be fully true). Given that we believe $Fe(\overline{n}) \rightarrow At(Fe(\overline{n+1}))$, when we add $Fe(\overline{n})$ to our stock of beliefs, $Fe(\overline{n+1})$ seems highly acceptable. So $Fe(\overline{n}) \rightarrow Fe(\overline{n+1})$ passes the Ramsey test *to a very high degree* and hence it is *highly acceptable*. Thus, applying a general and useful approximation heuristic, we go along with it, at least initially. Of course later, when we see where things lead, we realise that this is a situation in which the heuristic leads to trouble, and so we do not have the considered intuition that the sorites argument is sound. When we withhold the approximation heuristic, the argument does not go through any longer.

2. We will argue that, for all its merits, the formalization in [4] does not really follow Goguen's proposal. Instead, we propose the following alternative formal argument:

$$\frac{\begin{array}{c} Fe(\overline{0}) \\ Fe(\overline{0}) \rightarrow Fe(\overline{1}) \\ Fe(\overline{1}) \rightarrow Fe(\overline{2}) \\ \vdots \end{array}}{At^{2^N} Fe(\overline{N})}$$

whose premises are intended to capture Goguen's idea of the implications $Fe(\overline{n}) \rightarrow Fe(\overline{n+1})$ being close to fully true but not completely. We will show that, similarly to the previous one, this argument is valid (we will derive the conclusion in the same axiomatic system of [4]) but has an obviously true conclusion that dissolves the paradox. We will also obtain this formalization from a weaker notion of 'almost true'. Finally, we will discuss an explanation of the initial sorites paradox following the ideas of [3], but with no need to appeal to the particularities of Łukasiewicz operations.

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Acceptable propositional normal logic programs checking procedure implementation

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Keywords: logic programs, acceptable logic programs, Haskell

Our aim is to implement in Haskell programming language the procedure of establishing if a given propositional normal logic program is acceptable in an efficient way. The Connectionist Inductive Learning and Logic Programming system ($C\text{-}IL^2P$) enables to perform a translation of propositional normal logic programs [3] into artificial neural networks and reverse—three-layered, feedforward artificial neural networks into normal logic programs [2].

Propositional normal logic programs are defined as sets of Horn clauses, i.e. structures labelled h_n of the following form:

$$a_i \leftarrow a_j, \ldots, a_k, \sim a_l, \ldots, \sim a_m \tag{6}$$

where a_i is called the *head* of Horn clause h_n and a_j, a_k, $\sim a_l$, $\sim a_m$ are *literals* that form the *body* of Horn clause h_n. Literals can be of the form of *atoms* or negated atoms and negation denoted by $\sim$ is understood as a *negation by finite failure*. We will refer to atoms from the body of a given Horn clause h_n that are not preceded with negation as $body^p(h_n)$ and atoms that are preceded with negation as $body^n(h_n)$. The set of all atoms occurring in a logic program is called a *Herbrand base* and denoted by $B_{\mathcal{P}}$.

Neural networks that result from the translation of normal logic programs model the way the semantics based on the *immediate consequence operator* works for those logic programs. The immediate consequence operator is a mapping: $T_{\mathcal{P}} : \wp(B_{\mathcal{P}}) \longrightarrow \wp(B_{\mathcal{P}})$, i.e. a mapping from subsets of the power set of the Herbrand base of a logic program into subsets of the power set of the Herbrand base of a logic program, defined as follows:

$$T_{\mathcal{P}}(I_{\mathcal{P}}) = \{head(h_i) \mid h_i \in \mathcal{P}, body^p(h_i) \subseteq I_{\mathcal{P}}, body^n(h_i) \cap I_{\mathcal{P}} = \emptyset\} \tag{7}$$

where $I_{\mathcal{P}}$ stands for the Herbrand interpretation, which is a set of atoms that are *true* and belong to the Herbrand base $B_{\mathcal{P}}$. A model for a given logic program is a fixpoint of the immediate consequence operator and can be found by applying the immediate consequence operator to the result of its previous application, where the starting point is the empty set [4]. The situation looks similarly when neural networks are concerned: a neural network which stabilise itself after setting all of the input neurons in the minimal state (what refers to the empty interpretation) has its output layer neurons activated in pattern that corresponds to the model found by the immediate consequence operator.

In some cases it is impossible to find a model for a normal logic program by means of the aforementioned procedure, because the immediate consequence operator traps in an infinite loop, e.g. $\mathcal{P} = \{a_1 \leftarrow \sim a_1\}$. Neural networks that result from the translation of such logic programs cannot stabilise itself either. However, there is a method that allows to check if the immediate consequence operator can reach a solution for a given logic program, and such logic programs are called acceptable [1]. The motivation that stood behind the creation of the definition of an acceptable logic program was different from the decidability whether the immediate consequence operator traps in an infinite loop or not. For that reason the procedure is very complicated, because it requires creation of many additional structures, and overall it is costly in terms of computational resources. In Order to establish whether a given logic program is acceptable or not we have to:

1. Create a directed graph $G_{\mathcal{P}}$ for a given logic program.
2. Create a set $Neg_{\mathcal{P}}$ which contains all atoms that are preceded with the negtion in the logic program $\mathcal{P}$.
3. By means of the graph $G_{\mathcal{P}}$ and the set $Neg_{\mathcal{P}}$ create a set $Neg^*_{\mathcal{P}}$ that contains all atoms from the logic program $\mathcal{P}$ that each atom form the set $Neg_{\mathcal{P}}$ depends on.
4. Create a logic program $\mathcal{P}^-$ which contains those Horn clauses from the logic program $\mathcal{P}$ whose heads belong to the set $Neg^*_{\mathcal{P}}$.
5. Create Clark's completion of the logic program $\mathcal{P}^-$, denoted by $\mathsf{comp}(\mathcal{P}^-)$.
6. Associate every literal from the logic program $\mathcal{P}$ with a natural number by means of the level mapping $|\cdot|$, where the value for an atom a_i is the same as the value for the negated atom $\sim a_i$.
7. Find an interpretation I, which is a model for the logic program $\mathcal{P}$ and when we restrict the interpretation I to atoms form the set $Neg^*_{\mathcal{P}}$, it is a model for the $\mathsf{comp}(\mathcal{P}^-)$.
8. The logic program $\mathcal{P}$ is *acceptable* w.r.t. $|\cdot|$ and the interpretation I if for every Horn clause from the logic program $\mathcal{P}$ of the form

   $$a_p \leftarrow l_{r_1}, \ldots, l_{s_n}$$

   the following occurs:

   $$|a_p| > |l_{q_i}| \text{ for } i \in [1, \overline{n}]$$

   where

   $$\overline{n} = \min(\{n\} \cup \{i \in [1, n] \mid I \not\models l_{q_i}\})$$

In addition, described procedure is vulnerable to the order in which literals from the body of a Horn clause occur, e.g. logic program of the form $\mathcal{P}_1 = \{a_1 \leftarrow \sim a_1, a_2\}$ is not acceptable, while $\mathcal{P}_2 = \{a_1 \leftarrow a_2, \sim a_1\}$ is. Therefore, if we want to check whether a given logic program that was obtained from a neural network is acceptable or not, we have to run the procedure for all possible combinations of literals from the bodies of its Horn clauses.

The goal of this work is to implement the procedure of checking if a given propositional normal logic program is acceptable in an efficient way. The improvements concern the model search for the Clark's completion of the logic program, the numbers generation in the level mapping and creating suitable models for a logic program that enhance steps following afterwards. The whole system is implemented in Haskell programming language and the description of the working implementation is also a part of the presentation.

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A Semantic Representation of Humans' Conceptions in Terminological Systems

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Keywords: Description Logics, Semantic Analysis, Formal Semantics, Conception, Logic and Cognition, Ontological Analysis of Conception

Description Logics are the underlying logics of terminological systems, see [3; 7; 10]. Description Logics (DLs) can be regarded as the most widely used knowledge (as well as ontology) representation formalisms in semantics-based systems. Most DLs are decidable fragments of Predicate Logic (PL). However, I shall emphasise that DLs are syntactically modal logic. More specifically, $\mathcal{ALC}$ (the prototypical DL Attributive Concept Language with Complements) is developed using modal logic **K**[1] (see [6]). In other words, $\mathcal{ALC}$ is a syntactic variant of modal logic **K**. DLs represent knowledge in terms of: (i) *concepts* (that are equivalent to unary predicates in PL), (ii) *roles* (that are equivalent to n-ary ($n \geq 2$) predicates in PL and can be either relations or properties), and (iii) *individuals* (that are equivalent to constant symbols in PL). The set of logical connectors in $\mathcal{ALC}$ is: { conjunction ($\sqcap$), disjunction ($\sqcup$), negation ($\neg$), existential restriction ($\exists$), universal restriction ($\forall$) }. In addition, $\mathcal{ALC}$ supports { tautology ($\top$), contradiction ($\bot$) } and 'atomic concepts' as well as 'atomic roles'.

In DLs, concepts are non-logical symbols and the main constructors of world descriptions. From the philosophical and cognitive perspectives, we do not really know what concepts are. However, we can interpret concepts as the "unity of the act of bringing various representations under one common representation", see [9]. We may also consider concepts the mental images of the world that are labelled by some linguistic expressions, see [8]. For logical and terminological assessments, we can assume that concepts are manifestable in the form of conceptions in order to be expressed and stated, see [5]. Therefore, the main assumption here is that one's descriptions are expressible based on his/her own conceptions of the world. In this research, constructed concepts (in one's mind) are interpreted as conceptual entities that are the basic materials of his/her meanings. Thereby I have hypothesised that concepts (as conceptual entities) are the main building blocks of meanings in a DL-based terminological system.

Suppose that we are going to focus on logical-terminological analysis of Martin's meaning of the concept 'Leaf'. It is presumable that Martin's mental constructed concept 'Leaf' can—based on his own conceptualisation—be manifested in the form of a conception in order to be ex-

[1] **K** was named after Saul Aaron Kripke, who is an American logician and philosopher. Kripke is well-known for his valuable works on the semantics of modal logic.

pressed. More specifically, relying on nominalism (see [1; 2]), concepts can be transformed into either linguistic expressions or symbols in order to be expressed and represented. For logical assessment of Martin's conception of leaf, the conceptual entity 'Leaf' and its interrelationships with any other conceptual entity can—by 'predication' (that is a function)—be assigned into logical symbols. In [4], I have argued that the predication of Martin's conception of leaf is concerned with the question of 'what is it to state something about that conception (of Martin)?'. Relying on such a heuristic question, a predication tackles to find an answer for describing and expressing the question of 'what is there for Martin's produced conception?'. The latter question focuses on the existence of Martin's conception of 'leaf', so that question deals with ontological analysis of his conception of leaf.

This research is concerned with ontological analysis of humans' conceptions of the world based on description logics. I will make a logical background for semantic representation and formal semantic analysis of conceptions. It will be taken into consideration that the phenomena of 'meaning' and 'conception' are strongly tied together. Accordingly, I will focus on specifying the possible conceptualisation of humans' conceptions regarding DLs' terminologies.

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Deduction as a factive hypothesis

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Keywords: deduction, inference, deductivity measures

The normative difference between deduction and induction does not imply that the real inferential processes of induction and deduction are equally distinct, both at a psychological and at a neural level. In the face of the question: are there really factive differences between the psychological processes of deduction and induction?, the recent literature offers all logically possible answers. One line of work, the most extended and akin to the traditional comprehension of deduction, understands that there are in fact factive differences between the processes of deducing and inducing [3; 4; 10; 11; 15] as a consequence of an alleged a priori distinction between necessary and contingent consequence extraction. It is remarkable that this perspective is compatible with rival conceptualizations of deduction and of normativity in general. In particular, both the counter-example conception of deduction as also the calculative conception both in their truth functional and probabilistic versions assume that there are factive differences between deductive and non-deductive inferences. Another line of research records gradual factive differences between deducing and inducing either in a positive [7; 8] or a negative direction. Finally a third approach assumes that there are no factive differences between mental or cerebral inductive and deductive processes, as explicit in the words of Oaksford [9]: "…tasks are not deductive in and of themselves. What function a task engages is determined by the empirically most adequate computational level theory of that task".

This state of the art is extraordinary and shows that conceptual and experimental elucidation is needed on the eventual existence of deductive inferences, even if it is one of the oldest issues in science. From a psychometrical perspective, four basic measures of reasoning (operational, content, instantiation and strategies) confirmed [15] inductive and deductive procedures offered an equivalent contribution to measure reasoning. On the other part, pointing in an opposite direction, it is not evident that those factive processes in which inductions and deductions are realized are alien to the normative properties exemplified in them. For example, there are extended and repeated evidences systematically linking deductively canonical reasoning with cognitive capacities such as intellectual ability [2], intelligence indexes [14], psycotechnic competence [5], emotion handling [1], academic excellence and healthy cognitive ageing.

From the conceptual perspective, the frontiers between normative (logical or probabilistic) and factive (psychological or neural) inference have been historically blurred [16] and additionally there are not only rival conceptions of deduction as a psychological process, but also fundamental disagreements on the specificity of deductive inference at a neural level [6]. Even more importantly, there are rival conceptions of normativity and its import on factive processes [12; 13].

This paper introduces a number of measurable deductive variables and components: logical validity, probabilistic validity, computability, integration, logical vs relational complexity, modality, and reviews their presence in the most employed reasoning tests and subtests in conductual and neural research. For any given component, we determine if it is explicitly assessed and in that case if it is measured or quantified. We also consider non explicit deductive components which have influenced on reasoning tests, even if they are not quantified, assessed or even mentioned.The objective of the review is to gather the basic arguments, data and verify if one and the same deductive phenomenon is present in normative, psychological and neural levels. In the conceptual front, two constraints are proposed to give specific content to the hypothesis of factive deduction:

- factive deductive features should be qualifiedly invariant under distinct cognitive formats, in particular under linguistic/verbal and plastic/visual formats,
- a measurable construct of deductivity should include variables such as logical validity, computability, integration, logical vs relational complexity.

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A logical characterisation of a human language phenotype finds a central role throughout cognition

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Half of the problem of understanding evolved cognitive phenomena is characterising the phenotype. In understanding the evolution of the human capacity for language, the phenotype is probably even more than half of the difficulty. This talk is about what happens when one takes just a subset of the phenomenon (human narrative discourse) and uses a nonmonotonic logic to characterise it as a species of planning [3]. This first of all changes ideas about the relevant phenotypes of the ancestors, and second, it redraws the boundaries of the human cognitive faculties that are involved.

[2] proposed a nonmonotonic logic at the heart of human reasoning—specifically the component of reasoning to an interpretation. It argued for a rethink of the biology of cognitive evolution in the mould of EvoDevo (Evolutionary Developmental Biology). This latter argument appeared in chapters 6 and 9. It contrasted the Chomskyan (and more broadly linguistic) conceptualisation of language that focusses on the structure (phonological, syntactic, semantic) of infinite sets of sentences, with the level of discourse processes at which language is a medium of social action, the basic action being the cooperative communication of the preferred model of the narrative to its recipient. This is the process of generating a new interpretation for a fragment of a language in the light of general knowledge recruited from semantic memory, through principles of cooperation. The nonmonotonic logic of this process is Logic Programming (LP). LP is also known as 'planning logic'. All that you need to know about EvoDevo for this talk is that it says that evolution takes place by tweaking complex systems within phenotypes through changes in control genes, thus transforming one complex phenotype into another complex phenotype. This is contrasted with the idea (common in psychology) that evolution works by adding new modules.

So planning, rather than communication, is our one-word characterisation of (this part of) the modern language phenotype, and this changes where we look for the origins of this part of language in our ancestors. With this radical change of conceptualisation, two broad questions open up: Which of our other cognitive capacities can be usefully construed as planning? And what were our ancestors' planning capacities that transmogrified into modern language? This talk will focus on the first of these two questions where recent psychological advances have made a redrawing of the map an exciting possibility [1].

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Inferential Erotetic Logic and Cognitive Speech Acts

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Keywords: inferential erotetic logic, speech act theory, cognitive speech acts, consequence relations

There are at least two interrelated conceptions of cognition. On one conception cognition is a mental or neural activity or process. On another conception cognition consists in the performance of certain (cognitive) speech acts within language. To elucidate the relation between the mental and the linguistic entities is the purpose of several research programmes. In what follows I will not contribute to this. However, I will presuppose that there are some interrelations which, to give an example, could be formulated thus: For someone to be *convinced* that something is the case, is often a precondition for her to *claim* that it is the case; and for someone to *infer* from accepted premises that something will be the case, is often a precondition for him to *form the expectation* that it will come to pass. The fist and the last italicized phrases can be taken to refer to cognition in the mental sense; the second and third italicized phrases refer to cognition in the linguistic sense. Relying on such bridge principles I will talk about cognition in the linguistic sense, exclusively. Acts like claiming and inferring I will count as *cognitive acts*.

There is a straightforward reading of standard declarative logic which renders this discipline as being concerned with cognition, i.e. with cognitive acts. The 'syntactically' defined relation $\vdash$ associates with each pair $\langle X, \phi \rangle$ in that relation a sequence of object language expressions (a derivation). In typical cases, some of the first elements in this sequence are assumptions of elements of X and the final element is an inference of ϕ. The intermediate elements of the sequence consist of more inferences and, possibly, more assumptions (which need to be 'discharged', however). Like inferring, assuming can be understood as a type of cogntive acts associated via bridge principles with mental processes (in this case, possibly, the activity of 'entertaining a thought'). In some frameworks further kinds of cognitive acts are accomodated, for example adducing truths.

This conception of declarative logic as being concerned with cognitive acts, suggests the following question: Which (meta-)relations in which other logical frameworks can be seen as similarly related to the performance of cognitive acts of which type? Obviously, asking consitutes a candidate type of cognitive acts (cf. [2; 4]). Can they be considered to be a conceptual basis for relations of evocation, erotetic implication, or similar relations of question logic (just like inferring is a basis for $\vdash$)? The school of Erotetic Inferential Logic (IEL; [3; 5; 6]) seems to present an approach which, on the one hand, includes a conception of evocation and implication and, on the other hand, sees these relations as associated with processes that may be taken to consist of what I call cognitive acts (cf. "inferential"). Hence: Can evocation, erotetic implication, and possibly other relations in IEL be seen as being based on the performance of cognitive acts like asking, inferred asking, and possibly more?

The paper will analyze the IEL definitions of evocation and erotetic implication as well as IEL's rule concept trying to arrive at a calculus that accomodates *evocation sequences* and *erotetic implication sequences* regulated by decidable rules akin to inference rules in natural deduction calculi. In the case of evocation the goal will not be attained. In the case of erotetic implication the goal will be attained. These results will be discussed with regard to the cognitive interpretations of the relation(s) furnished by the calculi. Can the speech acts that are regulated by the calculi be seens as what I proposed to call *cognitive acts* (including the property of being relatable to cognition in the mental sense)?

It should be noted that the approach could be pronounced constructivist although it stands in contrast to earlier constructivist approaches, specifically to [1] where other forms of erotetic consequence are defined within a related theory of cognition. In an attempt to systematize the results, I will propose a classification of consequence relations for question logics. On a first level one may distinguish consequence relations in question logic with regard to whether questions are admitted in the antecedent or in the consequent of the relation (or both) and whether a declarative part occurs in antecedent or consequent. On a second level the various types of consequence relations are sorted with regard to the involvement of rules of asking (rules of initial asking; rules of inferred asking; rules of question transformation), rules of answering (separate answering rules or rules for other cognitive acts; rules of permissive or obligatory answering), and subsidiary rules.

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Language as a tool: Acceptance-based Pragmatics

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Keywords: Pragmatics, Speech act Theory, rational model of dialog, belief versus acceptance, tool use

In this research, the communicative level of dialog or interaction is seen as the coconstruction of a linguistic tool by dialog partners. The background mental attitude enabling such a practical reasoning is acceptance [1; 2]. The distinction between belief and acceptance is here based on their functional role: truth-oriented versus goal-oriented [5]. In order to precisely define this new mental attitude which is acceptance, its relationship with belief and communicative action, this research aims at developing a rational model of dialog [6].

In the first part of this presentation, we will argue that this approach contributes to the explanation of human behavior by refashioning the notion of cooperative speaker and by respecting properties of co-constructed linguistic tools. In the second part of the presentation, we will focus on the specification of the notion of acceptance.

The point of departure is a listing of phenomena that fail to find an appropriate explanation in dialog modeling and Pragmatics generally speaking. Firstly, we will enlighten current considerations of cooperative speakers. These considerations fail in mixing reasoning-based approaches of perspective-taking and reuse-based approaches [8]. The key issue is to explain how human beings may rely on different perspectives (their own point of view, their addressees one or common/shared beliefs) or on existing linguistic tools built during the preceding interactions. The point is that compartmentalization is needed between linguistic choices and epistemic states of dialog partners. We will explain why Stalnaker's notion of accommodation and common ground [7] fails to capture these aspects and how acceptance-based approach succeeds. Secondly, we will focus on the properties of the results of successful interaction. The properties of co-constructed linguistic tools (conceptualization, linguistic choices, speech rate, etc.)—that are reused—do not match with a belief state but match with an acceptance state: partner-specific, time-limited, etc. Moreover, envisioning the communicative level of dialog as the co-construction of a linguistic tool by dialog partners is compatible with the consideration of the co-construction of "good-enough representations" [3]. This consideration is crucial for interactive construction of mutual understanding.

The distinction between belief and acceptance is based here on their functional role: truth-oriented versus goal-oriented, that is—by extension—encapsulating facts (declarative knowledge) versus tools (procedural knowledge). This consideration is the point of departure for specifying the semantics of acceptance and for an extension of Speech Act Theory. To do so, we will present preliminary elements based on the cognitive basis of tool use [4]. An illustration of key principles will be provided on the specific case of referential expression generation and interpretation.

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Many-valued logics and strategic reasoning

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Keywords: game theoretical semantics, argumentation games, strategic reasoning

Game theoretical semantics has proven to be a powerful tool for explicating various systems of non-classical logics. It replaces the reasoning about the logical notions like truth or provability with strategic reasoning in a certain argumentation games. In this talk we concentrate on the ways in which we obtain certain nonclassical many valued logics by modifying structure of the evaluation game for classical logic.

Evaluation game for the classical logic

The basic motivation for game theoretical semantics is a situation, in which two people disagree if a particular statement in a certain situation is true or not. In the process of argumentation they are transforming the initial complex statement to simpler statements until they reach a statement the truth of which is evident.

We model the process of argumentation by an argumentation game of two players concerning the value of a formula in the given model. The moves of the game reflect the logical structure of the initial formula and the game ends when the players reach an atomic formula the truth of which is directly given by the model.

The evaluation game for classical logic (see e.g. [3]) is formally defined as a game of two players Eloise and Abelard who disagree about truth of a (classical propositional) formula φ in a model M (consisting of an evaluation of atomic formulas). Eloise defends truth of the formula (she starts the game as Verifier), Abelard thinks that φ is false (he plays the role of Falsifier). Each position of the game is given by a subformula of the original formula and the moves consist of choosing which subformula the game continues with. Admissible moves in the current position are given by the main connective of the current formula:

- disjunction $\varphi \vee \psi$
  the (current) Verifier chooses if the game continues with φ or with ψ
- conjunction $\varphi \wedge \psi$
  the (current) Falsifier chooses if the game continues with φ or with ψ
- negation $\neg\varphi$
  the game continues with φ with the role of the players switched
- atomic formula p
  the game ends, if p is true in M, the current Verifier wins, otherwise the Falsifier wins

The win/lose in a particular play of this game is obviously strongly influenced by the ability of players to reason strategically. The definition of truth of a formula in the game theoretical semantics cannot be dependent on subjective factors, therefore it is identified with *existence of a winning strategy* for the initial Verifier of that formula.

Properties of the classical game
The evaluation game for classical logic has certain characteristic features:

- two players
- win of one player is a loss of the other one (win-lose)
- the game ends after a finitely number of moves (finite depth)
- each player knows his/her position in the game (perfect information)
- initial formula evaluated classically

It follows from Zermelo's theorem that any game with the first four properties is determined (one of the players has a winning strategy), hence the classical evaluation game gives us indeed bivalent logic. What if we relax some properties of the classical game?

Perfect → imperfect information: IF logic
In Independence-Friendly logics of Hintikka and Sandu (see [3; 4]) the condition of imperfect information of the semantics game is relaxed. The game is not determined any more, some formulas are neither True nor False—this leads to a three valued logic, with a third value *undefined*.

A natural generalization of this approach is to allow *mixed strategies* and define the value of a formula (evaluated in a finite model) as an equilibrium payoff of Verifier (cf. [4]). As a consequence the condition of a *win/lose* game is generalized to the one of *constant sum* game (with values between 0 and 1).

Win/lose → constant sum: weak Łukasiewicz logic
We might also keep perfect information and allow instead non-classical evaluation of atomic formulas in the whole interval $[0,1]$. This gives us a fragment of **Ł** called weak Łukasiewicz (Kleene-Zadeh) logic, where disjunction is interpreted as maximum and conjunction as minimum. Again as a consequence we get a constant sum game. (See e.g. [2]).

Classical → non-classical value of the initial formula
We might moreover require that not only atomic formulas, but also the initial formula is evaluated non-classically. This makes the evaluation of a current formula a part of the state of the game and the rules for connectives have to deal not only with choosing a subformula to continue with, but also with transforming the current evaluation. As a result we get full Łukasiewicz logic with two kinds of connectives (strong and weak version of conjunction and disjunction). Moreover the rules of the game nicely demonstrate the difference between these two kinds of connectives. (See [1] for details.)

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Part IV

Refutation Symposium 2018

Refutations in Logics Related to Johansson's, Nelson's, and Segerberg's

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Keywords: Johansson's Logic, Nelson's Logic, paraconsistency, decidability, refutation

We provide refutation systems for logics studied by Segerberg in [6]. These logics are extensions of Johansson's minimal logic **J** by axioms that are variations on some well-known principles (especially, *excluded middle* and *linearity*). For example, the logic **JK** is the extension of **J** by (K) $\neg p \vee \neg\neg p$. They are paraconsistent logics in the sense that the principle of explosion (I) $p \wedge \neg p \to q$ is not valid. The logics are interesting because of their semantic characterizations, which are interesting mathematical structures. But some of them have philosophical motivations as well, for example the logic **Le** of classical refutability (or Carnot's logic); see [2; 5]. $\mathbf{Le} = \mathbf{L}(\mathbf{2}') \cap \mathbf{L}(\mathbf{2})$, where $\mathbf{L}(\mathbf{2})$ is Classical Logic and $\mathbf{L}(\mathbf{2}')$ is the logic of (that is, the set of all formulas valid in) $\mathbf{2}'$. $\mathbf{2}'$ is obtained from the two-valued Boolean algebra $\mathbf{2} = (\{0,1\}, -, \cap)$ by replacing $-$ with t such that $t(0) = 1 = t(1)$.

Another interesting example is the logic $\mathbf{J}' = \mathbf{L}(\mathbf{2}') \cap \mathbf{JI}$, where **JI** is Intuitionistic Logic. $\mathbf{J}'$ is the greatest paraconistent analogue of Intuitionistic Logic; see [8]. (We say that a logic **L** is a paraconsistent analogue of **JI** iff $\mathbf{J} \subseteq \mathbf{L} \subseteq \mathbf{JI}$ and $\mathsf{I} \notin \mathbf{L}$.)

Our refutation systems consist of refutation axioms and refutation rules involving Mints-style normal forms; see [4]. Our systems are modifications of the refutation system for Intuitionistic Logic described in [7]. As applications, we obtain the following important results.

1. Efficient decision procedures for these logics. (In [6], in some cases, the question of decidability was left open.)

2. Refined semantic characterizations of these logics by finite tree-type frames. Such frames are built from the finite syntactic refutation trees, but they need not be trees. For example, the logic **JK** is characterized by frames of this kind with the property that the set of normal worlds has a greatest element.

Our results can be extended to certain Nelson's logics (with strong negation $\sim$) that are extensions of the logic $\mathbf{N4}^{\perp}$; see [5]. They are paraconsistent in the sense that ($\mathsf{I}^{\sim}$) $p \wedge \sim p \to q$ is not valid.

For example, let us say that a logic **L** is a paraconsistent analogue of **N3** (the intuitionistic logic with strong negation) iff $\mathbf{N4}^{\perp} \subseteq \mathbf{L} \subseteq \mathbf{N3}$ and $\mathsf{I}^{\sim} \notin \mathbf{L}$. It turns out that the logic $\mathbf{L}(\mathbf{3}) \cap \mathbf{N3}$

(where **L**(**3**) is the logic of **3**) is the greatest paraconsistent analogue of **N3**. The matrix **3** results from **2** by adding a new value $1/2$ and defining $\to$ as follows. $1 \to x = x$, $0 \to x = 1$, and $1/2 \to x = x$ for all x. Also, $-$ is replaced with $\sim$, where $\sim 1 = 0$, $\sim 0 = 1$, $\sim 1/2 = 1/2$, and the designated values are $1/2$ and 1. ($x \cap y = \min(x, y)$.) We remark that **L**(**3**) is a famous paraconsistent logic; see [1; 3].

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Proofs and refutations getting married

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In this talk I intend to discuss and promote an old idea that I proposed in my 1994 paper on "Refutation Systems in Modal Logic", viz. the idea of developing systems of deduction that combine standard deductive systems for derivation of validities with refutations systems deriving non-validities of a given logical system. Such combined systems of deduction can employ inference rules involving both derivable and refutable premises and conclusions. Typical examples of such rules are Modus Tollens (if $A \to B$ is a theorem and B is refuted, then A is refuted) and the Disjunction property (e.g. in Intuitionistic logic) stated as an inference rule: if $A \vee B$ is a theorem and A is refuted then B is a theorem. More examples (in modal logics) will be given in the talk, and some potential applications will be discussed.

Implementing refutation calculi: a case study

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Keywords: refutation systems, interpolation, classical propositional logic

The paper [1] describes a new method of finding interpolants for classical logic. In contrast to the more standard approaches, this method makes use of the results related to refutation systems. Broadly speaking, a refutation system is a collection of non-valid formulas (refutation axioms) together with (refutation) rules preserving non-validity. The paper considers one such system designed for classical propositional logic (see [2]). It contains two rules $\frac{G_1}{G}$ and $\frac{G_2}{G}$ which work in tandem allowing the elimination of l, l^* (a pair of literals) from the initial formula G, obtaining two smaller formulas G_1 and G_2. The proof of the interpolation theorem depends on special normal forms that are then decomposed using these rules with the process being repeated a number of times. Since this method of finding interpolants has the advantage of being constructive, the paper also contains an implementation of this method. It is based on the following algorithm (essentially, a repeated use of the proof of the theorem). The input consists of two formulas X, Y s.t. $X \to Y$ is a tautology, X is in a CNF and Y is in a DNF (with Y^* standing for Y changed into a CNF).

---

if X and Y have variables in common **then**
    create Y^*
    if The set of pairs of literals in $X, Y^* \to \bot$ is not empty **then**
        randomly choose one pair of literals (say l, l^*)
        create G_1
        create G_2
        create an interpolant of the required form, containing l and l^* together with the recursive calls using G_1 and G_2
    else
        if $\neg X$ is a tautology **then**
            interpolant is $\bot$
        else
            interpolant is $\top$
        end if
    end if
else

```
    if X is not empty then
        if Y is not empty then
            if ¬X is a tautology then
                interpolant is ⊥
            else
                interpolant is ⊤
            end if
        else
            interpolant is ⊥
        end if
    else
        interpolant is ⊤
    end if
end if
```

Although fully functional in other respects, the implementation presented in [1] was limited to the case of formulas with at most four variables. In this talk we present an implementation of the above algorithm for any number of formulas. It has been tested on 1000 randomly-generated formulas with at most 10 variables, 20 conjuntcs and 20 disjuncts. The results seem to confirm the tendency noticed in the previous experimental setting with fewer variables, namely that of a linear performance of the implementation (the relation between execution time and size of interpolant) with some results indicating a more complicated situation (the relation between execution time and size of the original formula), thus justifying the need for further analysis. In this talk we present the implementation, describe the experimental set-up and provide an in-depth analysis of the results.

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Refutation as falsification

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There is certainly more than one sense in which the term "refutation" is used in science. Refutation is often understood as a way of establishing non-validity, i.e. failure of being true in every model, but is may also be understood as demonstration of unsatisfiability, i.e., not being true in any model. Refutation may also be understood as demonstration of non-truth in a given model or demonstration of falsity in a given model, and, as is well known, in paracomplete and paraconsistent logics, a distinction may be drawn between non-truth and falsity. One and the same rule may be non-truth preserving and falsity preserving and therefore in one system be used to demonstrate non-truth, whereas in another system it may be used to establish falsity. In my talk I will present a system in which refutation is understood as falsification and where, moreover, refutation as falsification interacts with its dual, verification. I will mention some recent work by Sergey Drobyshevish showing that this interaction is in a sense essential to the system.

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From complementary logic to proof-theoretic semantics

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In the first part of my talk, I shall be concerned with $\overline{LK}$, a cut-free sequent calculus able to faithfully characterize classical (propositional) non-theorems, in the sense that a formula A is provable in $\overline{LK}$ if, and only if, it is not provable in LK (Tiomkin, 88; Goranko, 94). In particular, I will show how to enrich $\overline{LK}$ with two admissible (unary) cut-rules, which allow for a simple and efficient cut-elimination algorithm. I will then highlight two facts: (1) complementary cut-elimination always returns the simplest proof for any given invalid sequent, and (2) provable complementary sequents turn out to be *deductively polarized* by the empty sequent (Carnielli&Pulcini, 2017).

In the second part, I will provide a natural deduction system for complementary classical logic in terms of proof-nets and I will show how cut-elimination can be implemented directly on nets.

I will conclude the talk by observing how an alternative sequent system for complementary classical logic can be obtained by slightly modifying Gentzen-Schütte system $GS3$. This move could pave the way for a purely proof-theoretic account of multi-valuedness which, in turn, can be thought of as an alternative way to devise a proof-theoretic semantics for classical logic.

Applying the inverse method to refutation calculi

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The inverse method, introduced in the 1960s by Maslov [10], is a saturation based theorem proving technique closely related to (hyper)resolution [5]; it relies on a forward proof-search strategy and can be applied to cut-free calculi enjoying the subformula property. Given a goal, a set of instances of the rules of the calculus at hand is selected; such specialized rules are repeatedly applied in the forward direction, starting from the axioms (i.e., the rules without premises). Proof-search terminates if either the goal is obtained or the database of proved facts saturates (no new fact can be added). The inverse method has been originally applied to Classical Logic and successively extended to some non-classical logics, see, e.g., [1; 3–6; 9].

In all the mentioned papers, the inverse method has been exploited to prove the validity of a goal in a specific logic. We propose the dual approach, where the inverse method is applied to disprove the validity of a formula. We focus on Intuitionistic Propositional Logic (IPL) and we present a forward refutation calculus to derive the unprovability of a formula in IPL. Our motivation is twofold. Firstly, we aim to define a calculus which is prone to constructively ascertain the unprovability of a formula by providing a concise countermodel for it. Differently from backward proof-search methods, where rules are applied bottom-up, in forward proof-search the proved sequents need not be duplicated; accordingly, the obtained derivations contain few edundancies and the extracted countermodels are in general small. Actually, we can build a derivation of an unprovable formula so that the obtained countermodel has minimal height.

We also aim at clarifying the role of the saturated database yielded by a failed proof-search. In the case of the usual forward calculi for Intuitionistic provability, if proof-search fails, a saturated database is generated which "may be considered a kind of countermodel for the goal sequent" [11]. However, as far as we know, no method has been proposed to effectively extract it. The saturated database obtained by a failed proof-search can be considered as a kind of proof of the goal; we give evidence of this by showing how to extract from such a database a derivation witnessing the Intuitionistic validity of the goal.

The preliminary results of this research have been introduced in [7], where the forward calculus for IPL and the proof-search procedure are presented. An in-depth discussion of the method and new results (minimality of generated countermodels, extraction of derivations from saturated databases) can be found in [8]. To evaluate the potential of our approach, we have implemented the proof-search procedures in the prover *frj*, available at `http://github.com/ferram/jtabwb_provers/`.

We are investigating the applicability of the method to other non-classical propositional logics; so far we have considered the modal logic **S4** [2].

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From Łukasiewicz to Gentzen: On sequent-type refutation calculi for three-valued logics

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While traditional proof calculi deal with the axiomatisation of the set of valid sentences of a given logic, refutation calculi, also referred to as complementary calculi, are concerned with axiomatising the invalid sentences, i.e., providing means to deduce invalid sentences from already established invalid ones. Albeit already Aristotle studied fallacies (i.e., invalid arguments) in his system of syllogisms, the first modern treatment of axiomatic rejection originates with Jan Łukasiewicz [7] and his study of expressing Aristotle's syllogistic in modern logic [8; 9] where he introduced a Hilbert-type rejection system. This was continued by his student Jerzy Słupecki [16] and subsequently extended to a theory of rejected propositions [2; 15; 17; 18; 20]. (For a description of the development of this notion, cf., e.g., the paper by Wybraniec-Skardowska [21].) Furthermore, axiomatic rejection methods where not only studied for classical logic [13; 19] but also for varieties of logics, like intuitionistic logic [4; 10; 12], modal logics [5; 14], and others [1]. In this talk, I deal with the case of three-valued logics (and many-valued logics more generally), a logical approach again famously going back to Łukasiewicz [7]. While complete and uniform rejection methods were already quite extensively studied in the literature, like, e.g., in the works by Bryll and Maduch [3] and Skura [11], here I will discuss refutation calculi for such logics from the point of view of Gentzen-type systems. Furthermore, the relevance of the discussed calculi for nonmonotonic reasoning—notably for checking strong non-equivalence of logic programs under the answer-set semantics [6]—will be pointed out.

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A refutation calculus for intuitionistic logic

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Classically, logical consequence can be equivalently defined as truth transmission or as "backward" falsity transmission, in the following sense: that a consequence statement $\Gamma \vdash \Delta$ holds can be equated either with the condition that if all Γs are true, at least one of the Δs must be true as well, or, equivalently with the condition that if all Δs are false, at least one of the Γs must be false as well.

Intuitionism replaces the classical notion of truth with the constructive one of proof, where proofs are understood, via the Curry-Howard isomorphism, as computer programs: A consequence claim $\Gamma \vdash A$ is justified if there is an algorithm that taken a proof of each of the Γs as input produces a proof of A as output. Such algorithms are associated to deductions of Gentzen-Prawitz natural deduction systems. In particular to each rule for constructing deductions there is associated a basic operation, in the case of the implicational fragment the operations being λ-abstraction and function application:

$$\dfrac{\begin{matrix}[x : A] \\ t : B\end{matrix}}{\lambda x.t : A \supset B} \qquad \dfrac{t : A \supset B \qquad s : A}{\mathbf{app}(t, s) : B}$$

An inference rule is thus seen as encoding an operation that given proofs of the premises yields a proof of the conclusion.

In the present paper we show how the deductions of the implicational fragment of intuitionistic logic can be equivalently understood as computer programs that taken a refutation of their end-formula as input yield a refutation of (the conjunction of) their assumptions as output, and thus that also in the constructive setting it is possible, like in the classical case, to dualize the account of consequence by exchanging truth with falsity and reversing the direction of transmission.

Refutation is here understood as a primitive (metalinguistic) notion rather than analysed in terms of an (object language) operator of negation. At the level of atomic propositions, such an idea is not new (for instance, it is common to introduced a primitive predicate of apartness as the genuine constructive counterpart of negated equality). Here we propose to go one step further, giving a constructive meaning to the refutation of logically complex formulas starting from the refutation of their components.

To attain this goal, one has to associate to each inference rule particular operations that given a refutation of the conclusion yield a refutation of the premises. But what should these operations be?

$$\dfrac{\begin{matrix}[?:A]\\ ?:B\end{matrix}}{t:A\supset B} \qquad \dfrac{?:A\supset B \qquad ?:A}{t:B}$$

To answer the question, we do not start from the common idea according to which a refutation of $A \supset B$ is a pair consisting of a proof of A and a refutation of B. Though intuitive, this makes the notion of refutation depend on the one of proof. Our starting point is rather the idea that to refute a $A \supset B$ one has to show that it is impossible to convert a refutation of B into a refutation of A (this we take the closest way of dualizing the idea that a proof of $A \supset B$ is a function from proofs of A to proofs of B).

Taking this idea seriously, it follows that given a refutation t of B we have two alternatives (among which we may not be able to tell which is the case): either it is impossible to convert t into a refutation of A, or it is possible to do so, though we may not know how. We take the operation associated to the elimination rule to encode these two alternatives and thus as splitting the computation into two threads, one representing the alternative in which it is impossible to transform the refutation of B into one of A (that is, in which $A \supset B$ has been refuted), the other representing the alternative in which a refutation of A could be produced out of the refutation of B:

$$\dfrac{\mathtt{mkc}(t,x):A\supset B \qquad x_{\langle t\rangle}:A}{t:B}$$

The term $x_{\langle t\rangle}$ should be understood as having no real computational content, since we need not know anything about the refutation of A. In this sense it reminds of a variable, but it is not a real variable: we have not assumed to have a refutation x of A outright, but only postulated it as a possible alternative induced by the availability of a refutation t of B. (That x is not a real variable will be made precise below by taking the displayed occurrence of the variable x to be bound in $x_{\langle t\rangle}$).

The notion of impossibility underlying the informal explanation of the refutation conditions of $A \supset B$ is made precise in operational terms using the notion of error. Suppose that among the threads of a computation starting from a refutation x of B there are some that output refutations $s_1, \ldots, s_n$ of A. If one has a refutation t of $A \supset B$, one knows that these threads are spurious, i.e. that they represent impossible alternatives, since to have a refutation of $A \supset B$ is to know that it is impossible to convert refutations of B into refutations of A. Thus these threads can be closed with an error message that "explains" the incompatibility between the availability of a refutation of $A \supset B$ and the content of the thread (the possibility of producing—among other alternatives—refutations of A from a refutation of B).

We can thereby decorate the introduction rule for implication as follows:

$$\dfrac{\begin{matrix}\boxed{\mathtt{err}(t \mid x \mapsto s_i)}\\ [s_i^*:A]\\ x_{\langle t\rangle}:B\end{matrix}}{t:A\supset B}$$

where the error message $\mathtt{err}(t|x \mapsto s_i)$ plays the role of a discharge index in linking the assumptions with the inference rule at which they are discharged. The operational content of the rule is thus the following: when one has a refutation of $A \supset B$ one can reason as if one had a refutation of B (represented by the "merely stipulated variable" $x_{\langle t \rangle}$) with the additional information that subsequently generated refutations $s_i^* \equiv s_i[\![x_{\langle t \rangle}/x]\!]$ of A are spurious.

From these introductory remarks the following fundamental difference between the usual Curry-Howard correspondence and the correspondence between deduction and terms proposed in the present paper should be clear. In the usual Curry-Howard interpretation a whole deduction is encoded by the (unique) proof-term decorating the end-formula of the deduction, and the free variables of this term are those decorating the undischarged assumptions of the deduction. In the refutation-based interpretation we are proposing, a whole deduction is encoded by a family of terms decorating the undischarged assumptions and error messages associated to the discharged assumptions, and these terms and error messages will all depend on the (unique) variable decorating the end-formula of the deduction. Intuitively, each term and error message of a family should be thought of as corresponding to a thread of an algorithm that, taken a refutation of the end-formula as input, produces either a refutation of the (undischarged) assumptions or an error message as output. In general we are not in the position of selecting one thread as the one providing the "correct" output (for example, given a refutation of B we cannot tell in general, if we can obtain a refutation of A or of $A \supset B$).

It is worth remarking that to a deduction in which all assumptions have been discharged (i.e. a proof of the end-formula) we associate an algorithm which takes a refutation of the end-formula as input and outputs error messages from all of its threads. Informally, the algorithm shows that it is impossible to refute the end-formula. The analysis of "provability" as "impossibility of refuting" is closely connected to the analysis of classical provability via a double negation translation which lies at the basis of computational accounts of classical logic, especially of the $\lambda\mu$-calculus of Parigot [4]. In particular, in the context of the $\lambda\mu$-calculus, Crolard [3] defined two operations: `make-coroutine`(t, α) and `resume` t `with` $x \mapsto s$ and used them to formulate the Curry-Howard correspondence for subtractive logic (more commonly referred to as bi-intuitionistic logic) an extension of intuitionistic logic with a connective dual to intuitionistic implication.

The first author [1; 2] proposed several "parallel" variants of Crolard's calculus, as a term assignment to dual intuitionistic logic (and linear versions thereof), in which Crolard's operations were taken as primitive rather than defined.

The goal of our paper is twofold: First to provide a very concise and crisp presentation of the first author's "parallel" calculus, with the hope of making the main idea underlying it accessible to the widest audience possible. Second, we show that these operations can be used to develop a term assignment for refutations in implicational logic, rather than for proofs in subtractive logic as Crolard (and the first author) did.

After introducing the calculus by presenting terms, typing rules and conversions, we give a proof of strong normalization via an embedding of our calculus in λ-calculus. We close the paper with a comparison between our calculus and Crolard's original term assignment for subtractive logic, stressing the significance of the parallel nature of the calculus we propose.

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