

# Poznań Reasoning Week

*L&C 2016 | 14th ArgDiap | QuestPro 2016*

## ABSTRACTS

5–10 September 2016, Poznań

## About

In recent years we are witnessing a cognitive turn in logic. It results in inclusion of some areas of cognitive science, psychology and computer science into its hard core. Consequently, logic becomes capable of modelling actual cognitive activity of real life agents.

This turn does not create a rival for the mathematical logic: it forms a next step in the development of logic. It also reminds that for many centuries logic stood in a close and natural relationship to the science of actual reasoning processes.

Poznań Reasoning Week, consisting of three conferences, aims at bringing together experts from various fields, whose research focus on reasoning processes and their modelling from three perspectives:

- the interplay of logic and cognition (Logic and Cognition 2016);
- formal modelling of reasoning and argumentation (14th ArgDiap);
- natural question processing (QuestPro 2016).

## Invited speakers

(ArgDiap / L&C) *Ruth Byrne* (The University of Dublin)

(L&C) *Adam Chuderski* (Institute of Philosophy, Jagiellonian University)

(QuestPro) *Ivano Ciardelli* (Institute for Logic, Language and Computation, Universiteit van Amsterdam)

(QuestPro) *Jonathan Ginzburg* (UFR d'Études anglophones, Université Paris-Diderot)

(QuestPro) *Yacin Hamami* (The Centre for Logic and Philosophy of Science, Vrije Universiteit Brussel)

(QuestPro) *Philipp E. Korhonen* (University of Oxford)

(L&C) *Michiel van Lambalgen* (Institute for Logic, Language and Computation, Universiteit van Amsterdam)

(ArgDiap / L&C) *Keith Stenning* (Edinburgh University)

(ArgDiap) *Andrzej Wiśniewski* (Department of Logic and Cognitive Science, Institute of Psychology, Adam Mickiewicz University in Poznań)

## Acknowledgements

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**Part I**

**Logic & Cognition 2016**

# Invited talk: Counterfactual conditionals and the mental representation of possibilities

Ruth M.J. Byrne  
Trinity College Dublin  
University of Dublin, Ireland  
ruth.byrne@tcd.ie  
<https://reasoningandimagination.wordpress.com/>

I consider competing views of the nature of the mental representations and cognitive processes that underlie understanding and reasoning about counterfactual alternatives. One view is that counterfactuals are based on the computation of possibilities and another is that they are based on probabilities. I report experimental results on judgments of the truth of counterfactuals that indicate that people judge their truth based on the computation of possibility rather than probability. I also consider alternative views about the format of the mental representation of counterfactuals. One view is that it can incorporate symbols, e.g., a counterfactual to a set of facts can be represented by a symbol for negation, and another view is that it relies on embodied meaning, e.g., a counterfactual to a set of facts is represented by alternates. I report experimental results on inferences from counterfactual conditionals that indicate that counterfactuals can be represented symbolically rather than by embodied meaning.

## Invited talk: Relational reasoning in the human brain

Adam Chuderski  
Cognitive Science Department  
Jagiellonian University in Krakow  
adam.chuderski@uj.edu.pl  
<http://ecfi-group.eu/>

Relational reasoning refers to a general category of inferences that comprises many specific processes representing and transforming relations, like analogical, transitive, syllogistic, and spatial reasoning. Although different types of relational reasoning may rely on different cognitive mechanisms and strategies, psychometric studies have demonstrated that the individual scores on all these types of reasoning are strongly intercorrelated, and they converge to one reasoning ability factor (termed also fluid intelligence). This factor is crucial for describing human complex cognition, and it predicts not only other reasoning skills, like deductive or probabilistic, but also contributes to success in academic, professional, and daily life.

Unfortunately, despite more than half century of research on neurophysiological basis of reasoning ability, we still know little about brain processes carrying out relational reasoning (though we can quite precisely locate them in the frontal and parietal cortices). The individual differences in several candidate brain processes, or factors, like grey/white matter volume, neuronal speed, conduction reliability, signal power, and metabolic efficiency, explain an unsatisfactory portion of relational reasoning variance, too scarce to explain the phenomenon that some people excel in a wide variety of reasoning tasks, whereas other people fail to perform them.

In contrast, several formal models of relational reasoning have recently suggested that this process may be driven by so-called cross-frequency coupling (CFC) of neuronal oscillations. Specifically, premises and conclusions are represented and processed by both synchronous and asynchronous fast oscillations (usually, the gamma band) modulated by the phase of some slow oscillation (like the delta, theta, or alpha band). We present our computational model which explains how several types of relational reasoning can be realized by CFC. The model predicts that individual effectiveness of relational reasoning depends on the number of asynchronous fast oscillations that can be fitted in one slow oscillation (the fast-to-slow frequency ratio). We also present the first evidence from a EEG experiment on matrix reasoning showing that such a ratio strongly predicts the scores on a reasoning task, both intra- and interindividually.

In general, CFC may serve as the optimal level of description of complex neurocognitive processes, integrating their genetic, structural, neurochemical, and bioelectrical underlying factors with their explanations in terms of cognitive operations driven by neuronal oscillations.

## Invited talk: Kant, time and cognitive science

Michiel van Lambalgen  
Institute for Logic, Language and Computation  
Universiteit van Amsterdam  
M.vanLambalgen@uva.nl  
<http://uva.academia.edu/MichieltvanLambalgen>

The customary representation of time as the one-dimensional continuum of the real numbers is useless as a model of psychological time and its development. For example, the experimentally confirmed existence of the 'specious present' – an instant which has nonzero duration, but is still perceived as a unity – leads to a topology of time which is much stronger connected than that of the reals.

Another example, taken from developmental psychology: the order of events is sometimes encoded in the child's mind, but is generally not accessible to reasoning (e.g. children find it difficult to recite an event sequence in reverse order). The aim of this talk is twofold: firstly, to show that these phenomena (and more) can be profitably related to Kant's theory of time in the Critique of pure Reason, and secondly, that this theory can be given a mathematical form such that all of Kant's 'synthetic a priori principles' for time are validated. We close by relating the mathematical formalism to cognition and to contemporary debates on the nature of time.

# Invited talk: What is the working relation between intensional and extensional reasoning and judgement?

Keith Stenning  
University of Edinburgh  
k.stenning@ed.ac.uk

'Intensional blindness' characterises a phenomenon in the cognitive reasoning, judgment and decision making (JDM) fields' progress in the last half century. Experimenters assume that their subjects are trying to do extensional reasoning (and failing miserably) when subjects are actually, not unreasonably, engaged in intensional reasoning. [3] and [2] are two celebrated experiments which illustrate. [1] identifies Logic Programming (LP) as a suitable logic for modelling intensional reasoning in cooperative discourse, and sketches a multiple-logics approach which distinguishes reasoning *to* and *from* interpretations. Here we draw some corollaries from this approach in proposing an account of how LP can interface with probabilistic reasoning, so giving an example of how intensional and extensional reasoning can work together.

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# *Difficult deductions and fluid intelligence: Deductive Flexibility Test vs Raven's Advanced Progressive Matrices*

Kinga Antonik-Jonczyk  
Dajana Bieganowska  
Agnieszka Dubowska  
Dominika Koch  
Iga Kropa  
Maciej Małkowski  
Dawid Ratajczyk  
Anita Steć  
Natalia Żyłuk  
Institute of Psychology  
Adam Mickiewicz University  
d.bieganowska@gmail.com

## **Abstract**

We report research on correlations between fluency in difficult deductive reasoning and fluid intelligence, and also some other high-level cognitive abilities: need for cognitive closure and level of epistemic understanding. As we found no statistically significant results, our results are in line with previous research on this type of correlations. Additionally, psychometric properties of the methods used were mostly consistent with the ones presented in previous research, with one significant exception of The Standardized Epistemological Understanding Assessment.

**Keywords:** deductive reasoning, fluid intelligence, cognitive abilities

## **Extended abstract**

### **Objective**

Our main aim in this research was to test if there are any significant correlations between the cognitive abilities operationalised by means of results in Raven's Advanced Progressive Matrices (APM) and Deductive Flexibility Test (DFT) in young adults. APM results are generally thought of as a good measure of reasoning ability component of general intelligence, especially its fluid factor [1]. If untimed, this test is designed to differentiate between people at the high end of intellectual ability. When administered under timed conditions, the APM can also be used to assess intellectual efficiency [6]. DFT has been developed by Żyłuk and Urbański [10]

to account empirically for different ways of understanding the concept of deduction in psychological vs logical traditions. In DFT items the task is to choose all and only these combinations of premises which justify the conclusion. Thus the correctness of solution is based not only on ability to identify the entailment relation, but also employs reflective self-regulation, in particular with respect to metalogical norms [7]. Previous research [9] demonstrated, that fluid intelligence and fluency in deductive reasoning are correlated, as predicted by the CHC model of intelligence [2, ?]; however, this holds only for simple, or easy, deductions, as operationalised by syllogistic reasoning. In case of more complex deductions, operationalised by erotetic reasoning and polysyllogisms, subjects educational experience needs to be taken into account.

Additionally, as correctness of DFT solution requires substantial self-regulative ability, we were interested also in possible correlations between DFT results and results in tools measuring other high-level cognitive constructs: need for cognitive closure (NFC, [4]; in our research we employed shortened version of the scale, as described in [3]) and level of epistemic understanding (SEUA) [5, 11].

## Method

Four tests (APM, NFC, DFT, SEUA) were administered, in Polish, to 30 participants (18 women), students of different curricula aged 21 to 25, in two ca 45 min. sessions, separated by a 5 min. break.

## Results

There were no significant correlations between the tests' results. In most of the cases, we obtained satisfactory reliability levels (assessed using Cronbach's  $\alpha$  coefficient and Guttman's  $\lambda_2$ ). Also, reliability results were mostly consistent with those reported in previous Polish research employing tools used. One significant exception was SEUA, which was administered in a paper-and-pencil setting (as were the other methods). The results showed lower levels of reliability of this instrument than the ones obtained in an interview setting [11].

The obtained reliability levels were:  $\lambda_2 = 0.808$  for APM,  $\alpha = 0.662$  for NFC (between 0.613 and 0.754 for test's subscales),  $\lambda_2 = 0.804$  for DFT and  $\alpha$  ranged from 0.428 to 0.670 for SEUA's subscales (there is no general score for SEUA).

## Conclusion

In line with previous studies, our results confirm that the concept of deduction in CHC model, which stems in fact from logical tradition, is not fine-grained enough as to account for many manifestations of deductive reasoning. However, further research are needed in order to provide more detailed analysis of different kinds of deductive reasoning.

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# *On Logical Characterisation of Human Concept Learning within Terminological Systems*

Farshad Badie  
Center for Linguistics  
Aalborg University  
badie@id.aau.dk

## **Extended abstract**

In this research the phenomenon of 'learning' is seen as the process of [concept] construction, and thus, 'learning' will be assumed to be supported by an epistemology which argues that knowledge is generated (and constructed) from an interaction between human beings' experiences and over their conceptions, see [13], [5], [4], [15], [14]. This research focuses on analysing the logical characteristics of 'human inductive learning over her/his conceptions' within terminological systems. It focuses on applications of Description Logics (see [1]) and concept languages to the analysis of actual human inductive reasoning (and learning) in order to make a linkage between logic and cognition. The main objective is providing a logical background for theorising over the Human Concept Learning Problem (HCLP) in terminological systems.

My point of departure is the special focus on structural and analytical descriptions of 'concepts'. According to [2], concepts might be understood as representations of [aspects of] reality in mind. Considering concepts as mental representations, any concept could be considered to be equivalent to a psychological (and mental) entity<sup>1</sup>, see [12], [17]. Concepts as mental entities could be studied and analysed by the representational theory of the mind (and the theory of mental representation), see [16], [7], [8]. In this research the term 'concept' is suggested as following: a concept may be said to be a linkage and interconnection between the mental representations of linguistic expressions and other mental images (e.g., aspects of representations of the world, and of representations of inner experiences) that a human being has in her/his mind, see [6]. I shall claim that humans' conceptions are the outcomes of their constructed concepts. Let me offer an example: John has a visualisation of 'book' in his mind. Regarding his mental image, he may describe the concept of 'book' by 'set of written sheets'. Obviously, John's conceptions about book (based on his concept description) is transformable into a hypothesis like "book is a set of written sheets". The central word of this hypothesis is the distinct entity 'book'. Therefore, John has established the correspondence 'set of written sheets' to the entity 'book' (by means of the logical connector 'is a'), and thus, he has made a logical relation (and dependency) between a distinct entity and a description (or concept definition). In fact, he has made a mental assignment from the description 'set of written sheets' to the entity 'book'. Consequently, all characteristics, attributes and properties of 'set of written sheets' are corresponded

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<sup>1</sup>See 'The ontology of concepts' in: <http://plato.stanford.edu/entries/concepts/#ConMenRep>

to ‘book’. From John’s point of view, the applications of ‘book’ are determined and supported by the primary mental expression and definition “book is a set of written sheets”.

This research—with regard to the suggested realisation of concepts, mental concept representations and conceptions—focuses on analysing the logical characteristics of ‘human inductive<sup>2</sup> learning over her/his conceptions (as the outcomes of her/his represented concepts in mind)’ within terminological systems. It aims at providing a logical background for theorising over Human Concept Learning Problem (or the Problem of Human Concept Learning) in terminological systems. In fact, a terminological system—terminologically—expresses the representation of knowledge over concepts (in humans’ minds) within concept learning approaches. Let me be more specific; the most fundamental feature of human concept learning is using background knowledge, which is constructed and shaped over humans’ pre-constructed concepts, and, respectively, over their pre-shaped and pre-formed conceptions. In general terms—referring to [10]—background knowledge represents the knowledge of the world in general, or of the life in the specific society, the understanding of which people can be assumed to share as a framework for talking with each other. More specifically, in the learning sciences, background knowledge can be defined as the knowledge that learners have (or have learned) both formally in the learning environments as well as informally through their life experiences, see [9], [3]. I shall, therefore, claim that the background knowledge of an individual can be constructed based on (i) descriptions of the world grounded in pre-concepts (i.e., pre-concept descriptions) in the mind, and (ii) the conceptions that are generated with regard to the nature and the existential structures of the parts of reality in mind. However, there are strong dependencies between pre-concept descriptions (and pre-conceptions) and structural conceptions (of concepts). In fact, pre-concept descriptions could be realised to be the outcomes of structural conceptions (of concepts). Furthermore, the processes involved in human concept learning can be interpreted as an explanatory structural study [of the learner’s mind] and as a comprehensive and existential study [of her/his self]. According to the afore-mentioned characteristics of background knowledge, I define human concept learning as following: “Concept learning is an inductive learning theory that is logically shaped over a system of evidential support, which is structured over less-than-certain inferences. Concept learning focuses on conceptualisation of a concept through experiencing a set of examples (and a set of non-examples) of that concept”. In concept learning with background knowledge, a human being—with regard to the constructed set of experienced examples and over her/his background knowledge—focuses on hypothesis generation, see [11]. This research focuses on describing and analysing logical characteristics of hypothesis generation. Subsequently, a terminological basis for human concept learning will be provided.

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<sup>2</sup>An inductive logic is a system of evidential support that extends deductive logic to less-than-certain inferences (see <http://plato.stanford.edu/entries/logic-inductive>). The premises of a strong inductive argument should be capable of providing some degree of support for the logical conclusion, where such support means that the truth of the premises indicate, with some degree of strength, that the conclusion is (could be, could be capable of being) true. As evidence accumulates, the degree to which the collection of true evidence statements comes to support a hypothesis, as measured by the logic, should tend to indicate that (i) false hypotheses are probably false and (ii) true hypotheses are probably true.

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# *Subject's understanding of conditional sentences. Content analysis of Socratic dialogues during solving various versions of Wason selection task\**

Andrzej Gajda  
Szymon Chlebowski  
Mariusz Urbański  
Emilia Soroko  
Adam Mickiewicz University  
andrzej.gajda@amu.edu.pl  
<http://reasoning.edu.pl>

## **Abstract**

We present results of two studies on five versions of Wason selection task. The first study was a paper-and-pencil one, and the second was based on a standardized dialogue scenario. Results from the first study were significantly different from those obtained by Stenning and van Lambalgen [5]. Therefore, the aim of the second study was to deepen the understanding of the way the subjects reason while puzzling out five versions of Wason selection task.

**Keywords:** Wason selection task, conditionals

## **Extended abstract**

Wason selection task [6] is one of the most popular tools used in research on human reasoning. According to the original interpretation (Wason's own included) there exists only one correct solution to the task. The solution is based on classical understanding of the given rule (as a material implication  $p \rightarrow q$ ). The correct answer rate<sup>1</sup> for the original version of the task is very low (average is about 5 %) [5]. Stenning and van Lambalgen [5] proposed four different modifications of the original task: two-rule, contingency, truthfulness and conjunction, which were supposed to increase the correct answer rate.

We have carried out two studies. In the first study participants ( $N = 158$ ; 35 women, 119 men, 4 N/A; average age 20.03 years,  $SD = 0.62$ ) from Poznań University of Technology were

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<sup>1</sup>By the *correct answer rate* we mean the frequency of the solutions that are consistent with the interpretation of the task in the classical propositional logic, i.e. in case of the original task the selection of cards A and 7.

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Table 1: Correct answer rate for different versions of Wason selection task

<b>Version of Wason task</b>	<b>Our study [%]</b>	<b>S. &amp; van L. [%]</b>
<i>Classic</i>	12.12	3.7
<i>Two-Rule</i>	8.82	24
<i>Contingency</i>	3.23	18
<i>Truthfulness</i>	6	13
<i>Conjunction</i>	25	13

divided in five groups and assigned one version of Wason selection task each. Results of the first study are depicted in the Table 1.

The results of the first study do not match the results obtained by Stenning and van Lambalgen [5, p. 109]. Therefore we have decided to design the second study in order to deepen our understanding of the reasoning processes in all five versions of the task.

In the second study participants ( $N = 60$ ) were divided into groups: subjects who participated in a course of logic ( $N = 30$ ) and those who did not ( $N = 30$ ). Each group divided further on five versions of Wason selection task with two conditions: (1) presented cards supporting the rule given in the task vs (2) not supporting it.

Semi-structured interview (dialogue between experimenter and subject) was used to gather qualitative data. The structure of the interview was as follows:

1. *Preliminary conversation* — experimenter introduces the procedure of the study to the subject in two ways: the subject reads those informations and then the experimenter repeats them. Afterwards, the subject signs the consent for the participation in the research. At the end the task is presented to the subject.
2. *Hypothetical part* — experimenter lays down four cards and asks the subject for an answer with explanation. Additionally, experimenter asks subject about the information possibly gained from turning each card.
3. *Practical part* — subject turns selected cards and is asked about the answer.
4. *Revision* — the cards are removed from the table and subject is asked about the falsity criteria for the rule given in the task. Afterwards, the experimenter asks participant if she/he wants to change the previous choice of the cards.
5. *Classical interpretation* — the experimenter introduces to the subject the classical interpretation of the rule and asks about the choice of cards according to this interpretation.

The interviewers were the cognitive science students (2 persons), who were trained in applied logics and were aware of such processes as their influence on research participants as well as other social and emotional processes engaged in the course of the interpersonal contact. Therefore they were prepared not to interrupt the performance of the tasks. Moreover they used to make a verbal intervention only to move the reasoning forward or to reach the next stage of the interview. The interview was transcribed verbatim [4] and a deductive (*top-down*) content analysis was applied (see e.g. [2, 3]).

The second study (the outcomes of which are still being analysed) aims at revealing the kinds of reasoning behind the subject's answers. In order to realize this goal, the set of 27 categories of reasoning, that were justified theoretically, were established first. Categories were

divided into five groups: rule interpretation, assumptions, falsity criteria, experimenter role, dialogue structure. Categories were hierarchically ordered in a coding manual that was refined in a repetitive and recursive sample coding (3 interviews).

Second, when the work on coding manual has been finished (the tag system was repeatedly tested and covered the data in a sufficient way), the analysis with the use of three well-trained competent judges was performed. They coded the interview in order to identify different kinds of reasoning and to establish the frequency of the categories. The coders task was to interpret the participants speech as a manifestation of a distinctive reasoning strategy (e.g. [1]). The inter-rater agreement will be established using the appropriate inter-rater agreement coefficient. Finally, the every interview was coded according to the set of categories describing reasoning.

The tag system developed from the analysis of dialogues will be helpful in formal reconstruction of subject's reasoning patterns. The reason is that tags in the dialogues enable us to track the process of fixing the interpretation of the rule given in the task as well as the process of inferring conclusions from assumed interpretation.

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# *Abductive problems in a connectionist environment. Computational approach\**

Andrzej Gajda  
Adam Kupś  
Adam Mickiewicz University  
andrzej.gajda@amu.edu.pl

## **Abstract**

We define an abductive procedure which enables to generate abductive hypotheses in the general form of addition to, removal from or modification of information in the initial knowledge base. The Connectionist Inductive Learning and Logic Programming System (*C-IL<sup>2</sup>P*) for definite logic programs serves as a basis for the abductive procedure. We also present the implementation of the whole procedure in Framstics software system along with the application of the abductive procedure over few selected problems.

**Keywords:** Abduction, neural networks, Connectionist Inductive Learning and Logic Programming System

## **Extended abstract**

According to the algorithmic point of view abduction is a kind of reasoning leading to the formulation of hypothesis  $H$  which allows to *fill the gap* between a knowledge base  $\Gamma$  and a puzzling phenomenon  $\phi$ , unattainable from  $\Gamma$  [12, 7]. To be more precise, hypothesis  $H$  “is legitimately dischargeable to the extent to which it makes it possible to prove (or compute) from it as it is currently structured” [3, p. 88]. Such approach to the abduction can be found in [9] or [13]. In a slightly more general approach than the one offered by Gabbay and Woods one can understand an abductive hypothesis as a difference between the initial  $\Gamma$  and the final  $\Gamma'$ , where  $\Gamma'$  represents  $\Gamma$  *modified* by  $H$ . It could be the case that  $\Gamma'$  results from  $\Gamma$  not only by just *addition* of the new information but also by *removal* from or *modification* of some information in  $\Gamma$ . Our aim is to define abductive procedure which generate abductive hypotheses in the form of the difference between initial and final knowledge base (the term ‘difference’ can be understood here as a symmetric difference).

For this purpose we are going to use modified Connectionist Inductive Learning and Logic Programming System *C-IL<sup>2</sup>P* [4] for definite logic programs. It allows to represent  $\Gamma$  and  $\Gamma'$  as definite logic programs. The transfer from  $\Gamma$  to  $\Gamma'$  is obtained by three steps: (1) translation of  $\Gamma$

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and  $\phi$  into a neural network; (2) learning the neural network by means of the modified version of Backpropagation algorithm; (3) translation of the trained neural network back into the logic program  $\Gamma'$ .

Garcez et al. [5] already used *C-IL<sup>2</sup>P* to generate abductive hypotheses. Generally speaking, the two approaches presented in [5] focus on the search of the atoms that are 'responsible' for the activation of the atoms described as abductive goals. Our approach differs from those methods in several ways. As it was mentioned at the beginning, our abductive hypotheses can be represented by *removal* from or *modification* of information contained in  $\Gamma$  on the contrary to only *addition* of information to  $\Gamma$ . Another point is that the abductive hypotheses and abductive goals in approaches described in [5] are restricted only to the atoms. We extend the notion of the logical consequence of the definite logic program by Horn clauses. This move enables us to formulate abductive hypotheses and goals in the form of the Horn clauses as well.

The logical algorithms described above have been implemented using Framsticks software, a tool used to perform artificial life, but also computational logic experiments [10, 9]. The implementation encompasses the whole abductive procedure: translation of abductive problems into logical programs, translation of the logical programs into artificial neural networks, training of the artificial networks with examples containing information about abductive goal, translation of the trained networks into logical programs and reduction of the obtained programs by Quine-McCluskey algorithm [8, 11]. Apart from that, the general framework to perform computational experiments on abductive problems has been also implemented. This framework enables to test how the results of the abductive procedure depend on many parameters such as: number of neurons in the networks, thresholds and biases of the neurons, additional information contained in the training examples, particular characteristics of abductive problems, etc.

One of the important tasks of this work is to present results of the computational experiments concerning abductive problems. First, we will present results of abductive procedure for a number of selected problems. Then we will discuss the results of computational experiments concerning influence of selected parameters of networks and problems on the results of the procedure. The parameters selected are: number of additional hidden neurons, thresholds of additional neurons and size of the knowledge data base. The results will be characterized by features of the obtained abductive hypotheses, but also by the quality and stability of these abducibles.

Summing up, the presented abductive procedure allows to generate abductive hypotheses in the extended form, i.e. not only as addition of the information to the initial knowledge base  $\Gamma$ , but also as the removal or modification of the information contained in  $\Gamma$ . For the basis of our approach we used the modified version of *C-IL<sup>2</sup>P* System [4] which grants the representation of the initial and the final knowledge bases in the purely logical fashion, and in the same time it allows to benefit in the process of abductive hypotheses generation from the Backpropagation learning algorithm [2, 6]. The implementation of the abductive procedure was carried out in Framsticks software system [10]. We present the results of the application of the abductive procedure over few selected problems. Our future plans cover the extension of the abductive procedure for general and extended logic programs.

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# *A Non-Modal Deontic Logic*

Robert Kowalski  
Imperial College London  
rak@doc.ic.ac.uk  
<https://www.doc.ic.ac.uk/~rak/>

## **Abstract**

Deontic notions, such as obligations and prohibitions, are normally formalised in a modal deontic logic. I present a non-modal formalisation, in which obligations are goals in first-order logic, and the task is to satisfy the goals by generating some best model that makes the goals true. I will show how the approach deals with problems of contrary-to-duty obligations, which commonly arise in modal deontic logics.

**Keywords:** Deontic logic, Contrary-to-duty obligations, Preferences, ALP

## **1. Optimal goal satisfaction**

Formalising deontic notions in modal logic commonly encounters a variety of problems, including problems of contrary-to-duty obligations. I will argue that many of these problems can be solved using a non-modal approach that has many other applications, including combinatorial optimisation, abduction and defeasible reasoning.

A goal satisfaction problem is a tuple  $\langle M_0, G, W, \prec \rangle$ , where:  $M_0$  is a Herbrand interpretation (set of all atomic sentences true in  $M_0$ ) representing an incomplete history of the world;  $G$  is a set of sentences in FOL representing goals;  $W$  is a set of candidate Herbrand interpretations  $M \supseteq M_0$  representing alternative, more complete histories of the world; and  $\prec$  is a strict partial ordering between models, where  $M \prec M'$  means that  $M'$  is better than  $M$ .

A model  $M \in W$  satisfies  $\langle M_0, G, W, \prec \rangle$  if and only if  $G$  is true in  $M$  and no  $M' \in W$  is such that  $G$  is true in  $M'$  and  $M \prec M'$ .

## **2. Map colouring as goal satisfaction**

Consider a simple map with two adjacent countries *iz* and *oz* and two colours *red* and *blue*, where: It is obligatory to assign a colour to every country. It is forbidden to assign two different colours to the same country. It is forbidden to assign the same colour to two adjacent countries.

The problem can be represented by  $\langle M_0, G, W, \prec \rangle$ , where:

$M_0 = \{ \text{country}(iz), \text{country}(oz), \text{adjacent}(iz, oz), \text{red} = \text{red}, \text{blue} = \text{blue} \}$

$G = \{ \forall X [\text{country}(X) \rightarrow \exists C \text{ colour}(X, C)],$   
 $\forall X \ C1 \ C2 [\text{colour}(X, C1) \wedge \text{colour}(X, C2) \rightarrow C1 = C2],$

$$\forall X Y C \neg [adjacent(X, Y) \wedge colour(X, C) \wedge colour(Y, C)] \}$$

$$W = \{M_0 \cup \Delta \mid \Delta \subseteq A\}, \text{ where}$$

$$A = \{colour(iz, red), colour(iz, blue), colour(oz, red), colour(oz, blue)\}$$

$$\prec = \{\}, \text{ i.e. all models are equally good.}$$

The atomic sentences  $colour(X, Y)$  can be understood as describing actions.

There are exactly two models that satisfy the problem:

$$M_1 = M_0 \cup \{colour(iz, red), colour(oz, blue)\} \text{ and}$$

$$M_2 = M_0 \cup \{colour(iz, blue), colour(oz, red)\}.$$

The problem is more interesting if there are more countries and more colours, say three countries  $iz$ ,  $oz$  and  $az$  and three colours,  $red$  and  $blue$  and  $yellow$ , where  $iz$  and  $oz$  are adjacent,  $oz$  and  $az$  are adjacent, but  $iz$  and  $az$  are not. Suppose also that it is better to colour the map with as few colours as possible.

There are 12 models that make the goals true, but only 6 best models. But if we decide to colour  $iz$   $red$  and  $oz$   $blue$ , then we are obligated to colour  $az$   $red$ , because  $colour(az, red)$  is now true in the only best model remaining.

### 3. Goal satisfaction and deontic logic

The goal satisfaction approach can be viewed as defining a weak modal logic, in which goals  $p$  in FOL are implicitly prefixed with a modal operator,  $\mathbf{O} p$ , expressing that  $p$  must be satisfied. Viewed in this way, there are no nested modalities, and there are no mixed sentences, such as  $p \rightarrow \mathbf{O} q$ .

In deontic logic, the main focus is on inferring logical consequences. But, in this approach, the focus is on goal satisfaction. Nonetheless, it is possible to build upon the non-modal deontic logic of Horty [5], and define:

$$\mathbf{O} p \text{ is a logical consequence of } \langle M_0, G, W, \prec \rangle \text{ if and only if}$$

$$p \text{ is true in all best models } M \in W \text{ of } G.$$

There is an obvious relationship with the possible world semantics of modal logics.  $W$  is like a frame of possible worlds. The extension of  $M_0$  to  $M$  is like the accessibility relation between possible worlds. The preference relation between models in  $W$  is like the preference relation in preference-based deontic logics, such as [4] and [9]. However, whereas [4] and [9] build the preference relation into the semantics of the modal logic, here the logic is classical FOL, and the preference relation is external to the logic. Moreover, while in modal logics actions and events are normally represented by labels on the accessibility relation, here they are represented by atomic sentences in the models in  $W$ .

### 4. Contrary-to-duty obligations

Contrary-to-duty (CTD) obligations arise when a primary obligation  $\mathbf{O} p$  is violated and a secondary obligation  $\mathbf{O} q$  comes into force. In the goal satisfaction approach, this means that the real goal is  $p \vee q$  (equivalently  $\neg p \rightarrow q$ ), and that models in which  $p$  is true are preferable to models in which  $q$  is true. With this preference,  $\mathbf{O} p$  is a logical consequence of the goal  $p \vee q$ .

The most well-known example of the problems with CTD obligations is the Chisholm Paradox [2]: It ought to be that Jones goes to assist his neighbours. It ought to be that if Jones goes, then he tells them he is coming. If Jones doesn't go, then he ought not to tell them he is coming. Jones doesn't go.

Much of the discussion concerning the Paradox concerns conditional obligations of the kind involved in the second and third sentences. For example, the second sentence can be represented

either as  $\mathbf{O}(go \rightarrow tell)$  or as  $go \rightarrow \mathbf{O} tell$ . Different representations lead to different problems. See, for example, [1].

The problem can be represented by  $\langle M_0, G, W, \prec \rangle$ , where:

$$M_0 = \{\}, G = \{go \rightarrow tell, \neg go \rightarrow \neg tell\},$$

$$W = \{\{\}, \{go\}, \{tell\}, \{go, tell\}\} \text{ and } M \prec M' \text{ if } go \notin M \text{ and } go \in M'.$$

Both  $M_1 = \{\}$  and  $M_2 = \{go, tell\}$  make  $G$  true. But  $M_2$  is better than  $M_1$ . So both  $\mathbf{O} go$  and  $\mathbf{O} tell$  are logical consequences of  $\langle M_0, G, W, \prec \rangle$ .

Suppose, however, that we observe that Jones doesn't go. Treating observations abductively as goals, we update the problem to  $\langle M_0, G \cup \neg go, W, \prec \rangle$ . The only (and best) model that satisfies the updated problem is now  $M_1 = M_0 = \{\}$ , which implies the logical consequences  $\mathbf{O} \neg go$  and  $\mathbf{O} \neg tell$ .

## 5. Abductive logic programming (ALP)

The non-modal approach sketched here is an abstraction of the ALP [6, 8] approach to deontic logic in [7]. In [7], models are inductively defined by logic programs of the form  $conclusion \leftarrow condition_0 \wedge \dots \wedge condition_n$ , where  $conclusion$  and each  $condition_i$  is an atomic formula, and all variables are universally quantified. Any logic program of this form has a unique minimal model [3].

In [7],  $M_0$  is defined by a logic program  $P$ , and the models  $M \in W$  are defined by logic programs  $P \cup \Delta$  where  $\Delta \subseteq A$ , and  $A$  is a set of atomic sentences representing candidate assumptions. In ordinary abduction, the goals  $G$  represent a set of observations, and  $A$  represents hypothetical events that might explain  $G$ . In deontic applications, the goals  $G$  represent obligations and prohibitions, and  $A$  represents actions that might satisfy  $G$ . In applications to defeasible reasoning,  $A$  represents assumptions that conditions are normal, and  $G$  represents constraints that ensure conditions are not assumed to be normal if they are abnormal.

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# *The Majority Rule or the Equate-to-differentiate Rule? The Moderating Role of Regulatory Focus, Self-construals, and Culture Differences\**

Yong Lu  
Faculty of Theology  
Cardinal Stefan Wyszyński University in Warsaw  
luyong@student.uksw.edu.pl  
<http://www.lu-yong.com>

## Abstract

The majority rule and the equate-to-differentiate rule are two contradictory, albeit similarly structured judgmental heuristics. The present paper proposed the following theoretical deduction on the moderation role of self-construals, regulatory focus, and culture differences which affects decision makers to use the alternative rules when choosing between weak dominant pairwise options: Increasing the accessibility of the interdependent self-construal or of the information with a prevention focus or of intra-national culture identification causes individuals to be likely to use the majority rule, whereas increasing the accessibility of the independent self-construal or of the information with a promotion focus or of inter-national culture identification causes individuals to be likely to use the equate-to-differentiate rule.

**Keywords:** The majority rule; The equate-to-differentiate rule; Self-construals; Regulatory focus; Culture differences

## 1. The Majority Rule versus the Equate-to-differentiate Rule

Zhang, Hsee and Xiao focused on a decision heuristic named *the majority rule* for choice between binary multi-attribute options: the *majority-weakly-superior* option (slightly more favorable on most of its attributes) and the *minority-strongly-superior* option (considerably more favorable only on less of its attributes). The majority rule posits that the majority-weakly-superior option will be chosen. Furthermore, the regulation factors such as respond mode (choice vs. rating), information presentation (by cue vs. by option), and regrouping influence decision makers' choosing to use the majority rule [9].

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On the other hand, another contradictory, albeit similarly structured judgmental heuristic named *the equate-to-differentiate rule* proposed by Li states that decision maker equates one or several smaller different attributes of options and leaves the most distinct attribute as the assessment of the final choice [7].

Although both overall experimental results of the majority rule and the equate-to-differentiate rule provided a credible evidence on the underlying process of decision making (e.g., [8]), the obvious contradiction between the majority rule's preference to majority-weakly-superior option and the equate-to-differentiate rule's favorite to minority-strongly-superior option still opens a question on the two rules' underlying moderating conditions for application.

The present paper theoretically probed the moderating effect of the framing of the regulatory focus (prevention versus promotion), self-construals (interdependent versus independent), and culture differences (inter- versus intra-national) scheme that may induce decision makers to use the alternative of the two rules in choosing between weak dominant pairwise options. In the following, a specific deduction was proposed.

## 2. The Moderating Effect of Regulatory Focus, Self-construals, and Cultural Differences

Regulatory focus refers to the extension of *promotion* toward any specific goal, such as the pursuit of gains and aspiration toward ideals and hedonic pleasure, and *prevention* for losses and the fulfillment of obligations. A burgeoning literature has demonstrated the impact of distinctly motivational patterns involved in self-regulatory goals on such as behavioral strategies [5] and information processing [1]. Promotion versus prevention patterns can be made temporarily accessible in situations which emphasize gains achieved or losses avoided [2]. Furthermore, studies reported that individuals with a dominant *independent* self-construal tend to exhibit a bias toward promotion focus, weighting gain-framed information as more important, whereas those with a dominant *interdependent* self-construal tend to exhibit a bias toward prevention focus, weighting loss-framed information as more important [3][6]. Therefore, information framed in promotion terms may induce individuals to construct independent self-construal, whereas information framed in prevention terms may induce individuals to construct interdependent self-construal.

On the other hand, because the goal of interdependence appropriates for the majority's agreement, with an emphasis on maintaining relatively collectivism, the interdependent self-construal may be much more consistent with the assumption of the majority rule. On the contrary, because the independent self-view of being positively distinctness, with an emphasis on individualism, the independent self-construal may be more consistent with the assumption of the equate-to-differentiate rule. Furthermore, since that the majority rule per se focuses on collective superiority and that individuals from intra-national background manifest to collective tendency [4], those with intra-national culture identification may be much more apt to use the majority rule. Conversely, since that the equate-to-differentiate rule per se focuses on individual discrepancy and that individuals from inter-national background manifest to individual tendency [4], those with inter-national culture identification may be much more apt to use the equate-to-differentiate rule.

Taken together, the following deduction was submitted: Individuals who are more accessible to the information with a prevention focus or are manipulated to interdependent self-construal or are from intra-national culture are more likely to use the majority rule, whereas individuals who are more accessible to the information with a promotion focus or are manipulated to

independent self-construal or are from inter-national culture are more likely to use the equate-to-differentiate rule.

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# *On The Origin Of Metalogical Notions: The Case Of American Postulate Theorists\**

Jerzy Pogonowski  
Department of Logic and Cognitive Science  
Adam Mickiewicz University, Poznań,  
pogon@amu.edu.pl

## **Extended abstract**

We are going to discuss some results obtained by a few American mathematicians in the first three decades of the XXth century and published in the *Transactions of the American Mathematical Society*. The works in question are devoted to the foundations of mathematics and their authors are called *American Postulate Theorists*. The prominent among them are: Eliakim H. Moore, Oswald Veblen, Edward Huntington, Leonard Dickson. They have proposed several collections of postulates characterizing fundamental mathematical structures: groups, rings, fields, real numbers, complex numbers, systems of geometry (Euclidean and non-Euclidean, projective, etc.), algebra of logic. They were inspired by the earlier works by Peano, Dedekind, Pasch and Hilbert but they have developed their own style of dealing with foundational problems. Similarly to Hilbert, they have always tried to show that the postulates are mutually independent. Worth noticing is their choice of primitive notions, first of all in the case of systems of geometry. For example, Veblen used *points* and *order* (inclusion) as primitive terms, and Huntington used *spheres* and *inclusion* as primitive terms. However, the most important in their approach are the efforts to obtain uniqueness of the domains characterized by the postulates. Veblen has introduced the concept of *categoricity* and Huntington used the term *sufficiency* for the situation when a model is unique up to isomorphism. Both of them have articulated very interesting remarks concerning different forms of completeness, including a modest suspicion that syntactic and semantic aspects of completeness may not coincide. The following two quotations should clarify their position:

1. Huntington *A complete set of postulates for the theory of absolute continuous magnitude* (1902):  
The object of the work which follows is to show that these six postulates form a *complete set*; that is, they are (I) *consistent*, (II) *sufficient*, (III) *independent* (or *irreducible*). By these three terms we mean: (I) there is at least one assemblage in which the chosen rule of combination satisfies all the six requirements; (II) there is essentially *only one* such assemblage possible; (III) none of the six postulates is a consequence of the other five.

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2. Veblen *A system of axioms for geometry* (1904): [...] any proposition which can be made in terms of points and order either is in contradiction with our axioms or is equally true of all classes that verify our axioms. The validity of any possible statement in these terms is therefore completely determined by the axioms; and so any further axiom would have to be considered redundant. [Footnote: *Even were it not deducible from the axioms by a finite number of syllogisms.*] Thus, if our axioms are valid geometrical propositions, they are sufficient for the complete determination of euclidian geometry.

Notice that a precise metalogical notion of *completeness* was not well established at that time. Finally, they have also commented on Hilbert's *axiom of completeness* in geometry. This last topic is of special interest to us because the work on this paper is being sponsored by the National Scientific Center research grant nr 2015/17/B/HS1/02232 *Extremal axioms: logical, mathematical and cognitive aspects*.

The works of the American Postulate Theorists belong to the foundations of mathematics. They were not concerned with the distinction between syntax and semantics. With one exception (Huntington's remarks on Peano's notation) they did not refer to any system of formal logic. One has to remember that their works originated before formal logic has influenced foundational research: at that time only systems of Peano and Frege were known, and *Principia Mathematica* was under preparation. The results obtained by Löwenheim and Skolem also belonged to the future, though very near. The hope for arriving at unique characterizations of some fundamental mathematical structures could be thus seen as justified at that time. The situation has been then clarified by several limitative theorems obtained a little bit later.

The American mathematicians mentioned above are well known not only as creators of the systems of postulates: each of them has made his name for some important work in algebra, analysis, geometry and topology. It would be an exaggeration to call them a school. However, their works from the discussed period were influential. Most notably, the works of Huntington and Veblen were appreciated by Alfred Tarski who has crucially contributed to metalogic from the beginning of his academic activity. David Hilbert, Abraham Fraenkel, Rudolf Carnap and others tried to characterize the notion of completeness even before the birth of model theory. Then completeness and categoricity (categoricity in power) were intensively investigated in the classical as well as in the modern model theory.

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# *Cognitively Relevant Formal Properties of Diagrams*

Atsushi Shimojima<sup>1</sup>

Dave Barker-Plummer<sup>2</sup>

<sup>1</sup>Faculty of Culture and Information Science,  
Doshisha University, 1-3 Tatara-Miyakodani,  
Kyotanabe, 610-0394 Japan  
ashimoji@me.com

<sup>2</sup> CSLI/Stanford University  
Cordura Hall, 210 Panama Street  
Stanford, California, 94305, USA  
dbp@stanford.edu

Reasoning with diagrammatic representations has been studied both from logical points of view (e.g., [13, 15, 29]) and from cognitive points of view (e.g., [9, 12, 14]). Although these two approaches have a great potential for productive integration [30, 31], they have never been so integrated. In our view, a part of reason for this is that traditional logical studies of diagrammatic representations have not paid adequate attention to formal properties of diagrammatic representations that have direct implications on their cognitive functions. We believe that an adequate characterization of these formal properties can inform the construction of an experimentally testable psychological model of how different types of diagrams intervene in human inferential processes.

We report here on our logical approach to diagrammatic representation systems that serves to identify cognitively relevant formal properties of diagrams. We state conditions in which diagrammatic systems (1) amplify their own semantic coverage by supporting meaning relations that go beyond their basic semantic conventions, (2) afford automatic expression of consequential information upon the expression of a certain set of premises, and (3) force selective expression of non-consequential information upon the expression of a certain set of premises.

All these properties are cognitively relevant formal properties of diagrams. Property (1) has implications on how people read a richer set of information directly from diagrams through the process of “direct translation of perceptual patterns” [20] or “global reading” [21]). Property (2) concerns the perceived efficiency of deductive inferences based on diagrammatic representations [22] and has been shown to be functional in human inferential processes [27]. Property (3) is a negative aspect of diagrammatic representation, which leads people to produce representations containing overly specific information [32], so-called “accidental features” [5, 10] that invite erroneous inferences.

In investigating these properties, we take a *generic* approach: giving an abstract formal characterization of a class of diagrammatic systems, and then proving results about the semantic properties of all members of the class in the abstract setting. By adopting this approach, we are able to short-circuit investigation of individual diagrammatic systems, and assign the responsibility for the possession of properties of an individual system to its membership in the

class. We start by defining the general class of representation systems on the basis of channel theory, a formal framework for modeling information flow, of which representation is a special case, [8]. We then focus on a particular class of representation systems, Single Feature Indicator Systems [26]. SFISs are among the simplest representation systems that we can think of, and are built into a number of important, familiar diagrammatic representation systems, including bar charts, connectivity graphs, tables, and Venn diagrams. We demonstrate that they all share properties (1)–(3) above. In this way, we can explain why diagrammatic systems featuring visually quite different appearances have commonalities in their ways of both facilitating and hindering our inferential processes.

This approach can be contrasted to more individualistic approaches taken by a more traditional logical approach, such as work on Venn diagrams [28, 29], Euler diagrams [19], spider diagrams [15], Hyperproof diagrams [7], geometry diagrams [18] and dot diagrams [16]. Although this work has made significant contributions to our understanding of logical properties of these systems, the approach has been largely individualistic, in that semantics has been specified for each individual system and its logical properties have been investigated on the basis of the semantic specification dedicated to that system. It therefore has been difficult to see and explain the commonality of multiple systems—what, if any, features are shared by the ways representations in these systems convey information, support inferences, and otherwise facilitate our understanding.

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# *Abduction: some conceptual issues*

Mariusz Urbański  
Department of Logic and Cognitive Science  
Institute of Psychology  
Adam Mickiewicz University  
murbansk@amu.edu.pl  
<http://mu.edu.pl>

## **Abstract**

In this paper I explore differences and similarities between three classes of models of abductive reasoning: explanatory-deductive, explanatory-coherentist and apagogical ones, in particular with respect to criteria for evaluation of abductive hypotheses.

**Keywords:** abductive reasoning, explanatory abduction, problem-solving

## **Extended abstract**

What I shall argue for in this paper is that in research on abduction the very basic concept of abductive reasoning is constructed rather than described or even explicated. Such different constructions usually satisfy some common constraints of Aristotelian or Percean provenience, but do so in different ways and to different degrees.

In his reflection on abduction C. S. Peirce [6] famously went from early ‘syllogistic’ theory, in which abduction is seen as one element of a tripartite division of reasoning along with deduction and induction, to late ‘inferential’ theory, in which there is much more to abduction than just finding missing premise to produce a valid syllogism [1]. On this second view abduction is a complex form of reasoning, one of the most fundamental cognitive processes, allowing for succesful interpretative interactions with the physical world and other minds as well. However, under inferential theory all we can decisively claim about logical structure of abduction is that *affirming the consequent* has something to do with it.

In case of deduction we have at our disposal sound intuitions concerning what constitutes deductive reasoning and also generally we are able to effectively operationalize criteria for testing deductiveness. In case of abduction, however, we have intuitions only. Depending on answers to the following two questions: (1) Is abduction intrinsically explanatory? (2) If so, is this explanation of deductive character? existing accounts on abduction can be broadly divided into three classes of models: explanatory-deductive, explanatory-coherentist and apagogical ones [7]. What is common for all of them is that under each model the aim of abduction is to make sense of some puzzling phenomena. What does it mean to ‘make sense’ and what counts as ‘puzzling’ remains debatable.

I shall explore differences and similarities between these three classes of models, in particular with respect to criteria for deciding what makes one instance of abductive reasoning, or a hypothesis, better than the other. I shall draw on research on abduction in a computational setting of Inferential Erotetic Logic (multicriteria dominance relation as a basis for evaluating abductive hypotheses generated by a procedure based on Synthetic Tableaux Method [5, 4], a procedure for generating abductive hypotheses in the form of law-like statements based on Socratic transformations [8]) as well as in the context of experimental data on problem-solving (exploration of solutions to Raven's Advanced Progressive Matrices test [3] and to *Mind Maze* game [10, 9, 2]).

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# *Can Bayesian models have “normative pull” on human reasoners?\**

Frank Zenker  
Lund University  
Dpt of Philosophy and Cognitive Science  
frank.zenker@fil.lu.se  
<https://lu.academia.edu/FrankZenker>

## **Abstract**

Rejecting the claim that human reasoning can approximate generally NP-hard Bayesian models—in the sense that the mind’s actual “computations” come close to, or be like, the inferences that Bayesian models dictate—this paper addresses whether a Bayesian model can have normative pull on human reasoners. For such normative pull to arise, we argue, a well-defined and empirically supported approximation relation is required—but broadly absent—between (i) human reasoning on the ground and (ii) the behavior of a non-NP-hard model. We discuss a responsible stance on this issue.

**Keywords:** Bayesian inference, normativity, approximation

## **1. Extended abstract**

Cognitive science has been moving away from logical models, towards probabilistic ones, particularly Bayesian models [7]. These models have generally featured in accounts of human reasoning at the algorithmic level [11]. Regular slippage between Marr’s three levels [12], however, may have contributed to the virulence of the claim, that Bayesian models have a normative pull on actual reasoners, and so may serve as a normative yardstick for rational human behavior. The slippage here is between the use of rational in Anderson’s (see [1]) program for rational reconstruction [4], where rational denotes behavior adapted optimally to an organism’s representation of a task and an environment, on one hand, and the more traditional, fuller sense of *rational*, namely: normatively correct inferential choice-behavior, on the other.

This latter sense is invoked, for instance, when preferences are said to remain transitive and thus save agents who (must) choice-behave from being Dutch-booked. Since choice broadly includes choosing what to believe and how to act, the slippage also occurs in recent work on reconstructing and evaluating natural language argumentation (e.g., [8]; [6]; [5];[19]). Slippage may also have productive aspects, of course. But it remains at best unclear how the normative pull of a Bayesian model properly arises, if it does [18]; [13].

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The paper’s first part reviews the more technical side of an ongoing debate regarding the claim that human reasoning can approximate a Bayesian model, in the sense that the mind’s actual computations come close to, or may be similar to, behavior that the formal model dictates. Theoretical work in computer science shows that Bayesian reasoning is generally NP-hard [16], and can even remain so as the complexity of a Bayesian network is reduced. By contrast, under suitable assumptions – regarding the parameter space, the network size, and the relevance of intermediate variables, for instance - some Bayesian algorithms may deliver outputs in at time-scales that are realistic for human reasoners [9]; [10]; [15] [17]; [14]. But these assumptions neither pertain to Bayes’ update-rule nor to its role in making the network run. Therefore, some so-modelled episode of reasoning R1 may plausibly be an algorithmic-level candidate for a formal model of actual reasoning. But this may not hold for another such episode R2. What obtains, or not, thus depends on the particulars of a case.

While knowledge of the exact condition(s) under which a Bayesian model is not NP-hard is subject to ongoing research, extant results indicate that “smaller,” more “local” episodes of reasoning could plausibly be Bayesian inferences. How close outputs from such episodes then come to those delivered from heuristics is an open question (see [2]). It nevertheless seems clear that a responsibly meaningful sense of ‘approximation’ has not been forthcoming. Hence, the general claim that human reasoning may approximate Bayesian inference is currently not readily meaningful.

As the paper’s second part argues, on this background it may seem that Bayesian models of inference can after all have normative pull on human reasoners, namely for these “local” cases which are computationally tractable. For the sake of argument, however, one can even grant the (counter-factual) claim that a well-defined and mathematically demonstrated approximation relation holds between a computationally tractable model and an NP-hard model. A related question now is whether one has a well-defined and empirically supported approximation between human behavior on the ground and the behavior of this tractable model (Figure 1).

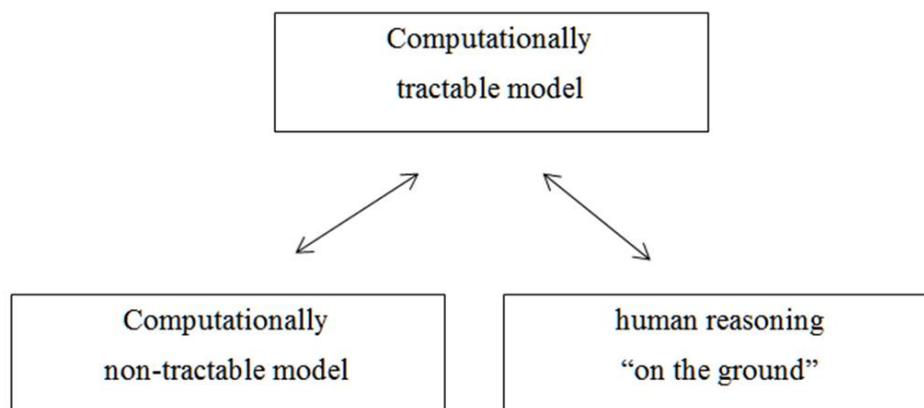


Figure 1: To grant the approximation relation between a non-tractable and a tractable model (left arrow) shifts the issue towards ascertaining whether an approximation relation holds between the tractable model and actual human reasoning (right arrow).

This new question obviously shifts the problem, and makes empirical facts relevant. Since computationally tractable models can easily, and massively, outrun what humans can do even

when they perform at their best, this places some weight on the ascertainability of an approximation relation between actual behavior and the tractable model.

The paper's third part hence discusses the reasonability of assuming that human reasoning can approximate the non-tractable model, an issue that [3, p. 56] has aptly summarized as follows:

“Eschewing the descriptive in favor of the normative does not erase all difficulties. For ‘ought’ is commonly taken to imply ‘can’, but actual inductive agents can’t, since they lack the logical and computational powers required to meet the Bayesian norms. The response that Bayesian norms should be regarded as goals toward which we should strive even if we always fall short is idle puffery unless it is specified how we can take steps to bring us closer to the goals”.

Rather than wishing to pass a final verdict, we primarily seek to make audiences (more) sensitive to this issue.

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# *Deductive Flexibility Test (DFT)*

Natalia Żyłuk  
Mikołaj Michta  
Institute of Psychology  
Adam Mickiewicz University  
nzyluk@gmail.com

## **Abstract**

Deductive Flexibility Test (DFT) was designed to assess the ability to choose all and only these combinations of premises which justify the specified conclusion. The pilot version of the tool consisted of 10 test items selected from the pool of 93. Each of the test items were created using classical categorical sentences. Based on the pilot study results, the decision about removal of 2 test positions was made. The final, 8-items version of the tool can be used in research on various cognitive abilities, in particular different types of intelligence.

**Keywords:** Deductive flexibility, deduction, syllogisms, intelligence, intelligence tests

## **1. Deductive flexibility**

In psychology, cognitive flexibility can be described as an ability to switch between thinking about different concepts and to think about multiple concepts simultaneously [5]. Notion of deductive flexibility was coined in analogy to cognitive flexibility, as an ability to determine different sets of premises, given which a certain conclusion follows. Although deductive flexibility could easily be characterized in terms of logic (referring to relation of logical entailment), its psychological operationalization requires further analysis.

## **2. The tool construction**

The tool was constructed by Żyłuk and Urbański [10]. Test items were created using classical Aristotelian categorical sentences of all the four types and on the basis of direct (operations of conversion, inversion, obversion, contraposition; rules of the square of opposition) and indirect (syllogisms) inference schemas. Building blocks of each of test items were 6 sentences – 5 placed above the line, served as a set of possible premises and 1 under the line, which was a fixed conclusion. In most of the cases, sentences above the line were selected in such a way that using some combination of them and the conclusion, one could build a valid schema of the inference (direct – with single premise or indirect – with two premises). Authors also created several tasks

without correct sets of premises. A single sentence from the set above the line could be used in different combinations of premises. Importantly, as a valid set of premises was considered only a “minimal” set, that is, a set that is sufficient to justify the conclusion.

Authors generated 93 test items (from 20 to 29 tasks for each of the four types of conclusion) using letters  $S, M, P$  as variables denoting classes of objects and letters  $a, e, i, o$  as infix operators referring to relations between such classes; two types of negation were also used in sentences: term negation ( $'$ ) and propositional negation ( $\neg$ ). The number of correct combinations of sentences from set above the line varied from 0 to 6. Given specific criteria, 10 test positions were chosen and authors then replaced variables present in them with pseudowords and phrases corresponding to possible relations between sets of entities that subject and predicative expression are referring to.

### 3. The pilot version of the tool

The set of 10 test items and instruction for participants were subject to evaluation. After applying changes suggested by the judges (including changing the order of tasks, the order of premises within certain tasks, correcting typos and short-cutting the instruction) and creating the scoring method, the pilot version of the tool was used in the study. The test was administered in Polish to 26 participants (14 women), students of different curricula with average age of 22.65. Given the small sample size, to assess the normality of test scores' distribution the Shapiro–Wilk test was used. The obtained scores had a normal distribution ( $W = 0.977, p = 0.794$ ). As the test items had a different difficulty levels, to assess the reliability of the tool, Guttman's  $\lambda_2$  [3] was used. The obtained reliability level was satisfactory ( $\lambda_2 = 0.893$ ).

### 4. The final version of the tool

In order to provide a tool suitable for use as a part of a larger battery of tests, authors decided to shorten the pilot version of DFT by removing 2 test items (criteria of removal: lowering the diversity of test items' difficulty levels, low diversity of inference schemas types used during item construction and low correlation between item and principal component performed in the course of factor analysis). To this day, the final version of DFT was administered in the first stage of study on relations between deductive flexibility, fluid intelligence [9], need for cognitive closure [7, 6] and level of epistemological understanding [11], and performed the reliability level above  $\lambda_2 = 0.800$ .

### 5. Possible applications

*Deductive Flexibility Test* is a thoroughly designed and reliable tool to measure what its authors defined as deductive flexibility. However, it should not be restricted to quantitative assessment of said ability. It appears that the novel structure of test items, where subject's task is to find smallest sets of premises that justify a given conclusion, makes it possible to use DFT as a tool to assess different strategies used by participants in solving such problems. Analysis of errors and patterns in which subjects choose and test sets of premises could give an insight into abductive reasoning processes.

## 6. Conclusions

*Deductive Flexibility Test* is a promising instrument with satisfactory reliability level. However, further research is needed in order to provide an adequate psychological operationalization of the "cognitive flexibility" notion; future research on relations between DFT scores and different aspects of cognitive functioning of an individual may help addressing this issue.

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**Part II**  
**14th ArgDiap**

## **Invited talk: Counterfactual thoughts and the controllability of events**

Ruth M.J. Byrne  
Trinity College Dublin  
University of Dublin, Ireland  
ruth.byrne@tcd.ie  
<https://reasoningandimagination.wordpress.com/>

People often create counterfactual alternatives to reality and imagine how the past could have been different 'if only...'. I discuss the ways in which counterfactual thoughts can help explain past events, for example, by identifying causal relations, and how they may also help prepare for the future, for example, by formulating intentions. Counterfactuals amplify emotions such as regret or guilt, relief or satisfaction, and they also support moral judgments such as blame, fault, and responsibility. I focus on research showing that people create counterfactuals by changing aspects of their mental representations of reality and that knowledge affects the plausibility of a counterfactual through a process of semantic and pragmatic modulation. I report the results of experiments that show that people focus on events within their control when they create counterfactual alternatives to reality, and I consider situations in which people imagine alternatives to events outside their control.

# Invited talk: Arguing with oneself, in a nonmonotonic logic, about a causal regularity's reliability

Keith Stenning  
University of Edinburgh  
k.stenning@ed.ac.uk

If we believe that at least several logics (or formal frameworks more generally) are required for analysing the many kinds of reasoning people undertake; and if we inhabit an era when reasoning, judgment and decision, and also argumentation have come to be treated extensively in probability, then the relation between probability and nonmonotonic logic as frameworks for cognitive modelling comes centre-stage.

[2] describes a reinterpretation of [1], and a Logic Programming model of its data, which we propose as a probability-free model of naive human justification of causal judgement: in particular, a model of how people retrieve from their memories, evidence of the reliability of everyday causal regularities, and base judgments on that evidence. A current experiment throws some light on the origins of data from which people reason, argue, judge, and decide. And raises some issues about the qualitatively different kinds of uncertainty that are in play in these mental processes.

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## **Invited talk: Deductive arguments and strong entailments**

Andrzej Wiśniewski  
Institute of Psychology  
Adam Mickiewicz University  
Andrzej.Wisniewski@amu.edu.pl  
<http://andrzejwisniewski.edu.pl/>  
<https://intquestpro.wordpress.com>

I will define and analyse a certain semantic relation between a set of declarative sentences and a declarative sentence, dubbed strong entailment. Although strong entailment is a subrelation of entailment classically construed, it is free of some of its drawbacks. I will argue that the concept of strong entailment is useful in argument analysis. In particular, it enables an explication of the intuitive notion of linked deductive argument

# *Annotating Judicial Decisions for the Sake of Identification of Case-Based Reasoning Structures in the Context of Statutory Interpretation\**

Michał Araszkiewicz  
Jagiellonian University  
Department of Legal Theory  
michal.araszkiewicz@uj.edu.pl  
<http://www.law.uj.edu.pl/ktp/pracownicy/ma/>

## **Abstract**

This paper presents a scheme for manual annotation of legal cases designed to identify arguments based on Case-Based Reasoning Structures in the context of statutory interpretation. The illustrative domain is the Polish law of protection of personal interests. Application of the annotating scheme leads to the development of a well-structured corpus of documents with potential utilization in information retrieval systems.

**Keywords:** annotation, case-based reasoning, statutory interpretation

## **Extended abstract**

There have been numerous studies related to identification of argument structures in common law jurisdictions (recently [5]) including research on identification of Case-Based Reasoning (CBR) structures ([7]). Less attention has been paid to the similar task in civil law countries. Professional lawyers are interested in retrieving a “smart” selection of legal cases which may be actually fruitfully applied in the current fact situation. This goal may be achieved also by annotating cases with respect to the important fact patterns which are present in them and which may promote or demote the strength of the claim of either party. Such fact patterns are notoriously discussed in the AI and Law literature under the label of factors and/or dimensions ([2], [1], [3] and many others, a recent more formalized approach [6]). The possibility to retrieve such smart selection of cases from a huge database could enhance the lawyers’ argumentative possibilities in given fact situations, as well as possibly lower the cost of legal service.

There is also a theoretical dimension to the task of annotating case law in civil law jurisdictions. Under civil law, the past cases often play a role of persuasive examples which may be, although not have to be, followed. However, in certain contexts such cases will be practically

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(although not formally) binding on courts and in certain procedural contexts they will be binding in a formal way, though not necessarily in identical manner as in common law jurisdictions.

This paper is a preliminary study on development of an annotation scheme for annotating judicial opinions in civil law culture context (where the Polish law serves as illustrative material), for the sake of identification and characterization of CBR structures used therein.

The domain chosen for the sake of this contribution is protection of the so-called personal interests. Personal interests are certain inalienable attributes of a person, such as in particular health, freedom, dignity, freedom of conscience, name or pseudonym, image, privacy of correspondence, inviolability of home, and scientific, artistic, inventive or improvement achievements (art. 23 of the Polish Civil Code, hereafter PCC). Personal interests are protected by civil law. According to the art. 24 of the PCC, a legal issue arises if a personal interest is threatened by another person. In such case, the person whose interests are threatened may demand the actions of another person be ceased. If the interests are not only threatened, but also infringed, a few other types of civil action arise. First of all, the plaintiff may demand that the person committing the infringement perform the actions necessary to remove its effects, in particular that the person make a declaration of the appropriate form and substance. If by means of an infringement a financial damage is caused, the plaintiff may demand the harm be compensated. Finally, if the infringement was done at fault, the plaintiff may demand a monetary compensation for the harm suffered or a fixed amount of money to be paid for a social cause chosen by the plaintiff. However, the claims of the plaintiff can be successful only if the behaviour of the defendant is not unlawful. The provisions described above create the core of statutory rules related to the protection of personal interests. There are basically two types of fact situations providing a ground for a claim (threat to or infringement of a personal interest) and three types of claims (for ceasing of activities, for removing the effects and for a monetary payment in three variants). The set of issues which have to be considered by the judge is dictated by the scope of the lawsuit as defined by the plaintiff and the arguments of the defendant. Depending on that, the following questions have to or may be considered by the court:

- 1) does the current fact situation involve any personal interest?
- 2) is there any threat or infringement of any personal interest?
- 3) do the arguments provided by the defendant justify the thesis that his actions were lawful?
- 4) was the harm done and was defendant at fault?

The above four questions pose the set of issues [4] which may be present in a given case concerning protection of personal interests. The statute does not provide for any clear criteria how these questions should be answered, because it was deliberately decided by the legislator to leave elaboration of these criteria to the judiciary. As the PCC has been in force for more than 50 years now, this resulted in development of a huge set of case law, including the opinions of district courts which are competent to hear these cases in the first instance, the appellate courts and the Supreme Court. The set of cases available in the commercial legal databases in this domain encompasses more than 900 decisions issued by appellate courts and the Supreme Court. For obvious reasons, this corpus is already too vast to be analyzed by a lawyer in a reasonable time. Therefore, the risk of missing adequate case law which could be rightfully used with respect to argumentation in the given fact situation is considerable.

The annotation scheme we are using has the following structure. It is composed of five parts:

- 1) important metadata,
- 2) procedural characterization of the case,

- 3) substantial characterization of the case,
- 4) relevant CBR structures,
- 5) characterization of argumentation.

Let us now present these parts in a more detailed manner. As regards the first part, the following metadata are annotated:

- a. the date of the decision,
- b. the name of the court issuing the decision and the division of the court,
- c. the signature of the decision,
- d. the composition of the panel of judges and name of the author of the rationale of the decision.

The metadata have the obvious function of individualization of the judicial opinion. The procedural characterization of the case encompasses the following elements:

- a. the instance of the court issuing the decision,
- b. the decisions of the courts of lower instances (if applicable),
- c. the procedural decision of the court,
- d. the indication of the winning/losing party.

The two categories of data indicated above are of lesser theoretical significance, although they are vital with regard to the operation of the information retrieval system. The third category, substantial description of the case, incorporates the following data:

- a. indication of the personal interest(s) in question,
- b. the type of claim(s) of the plaintiff and identification of issues,
- c. the factual grounds of the plaintiff' claim,
- d. generic characterization of the parties to the dispute (for instance: a politician, a journalist, a prisoner etc.).

The fourth part of annotation scheme concerns the CBR structures present in the rationale, including factors and dimensions. The pro-defendant or pro-plaintiff character of a factor (or direction of a dimension) is also indicated. The CBT structures are assigned to the issue to which they are relevant. The fifth category encompasses the types of arguments used in the argumentation of the parties and of the courts involved in the case. The arguments are divided into four following categories:

- a. rule-based arguments (top-level arguments used by plaintiff and defendant)
- b. classical canons of statutory interpretation,
- c. doctrinal arguments, and
- d. CBR structures
  - i. with references to past cases

- ii. without references to past cases.

Arguments based on CBR structures are then classified into those based on previous quoted cases and those that do not have a reference to previous case law. The latter arguments may involve development of new factors or consideration of new constellations of factors. The annotation scheme enables us to identify the types of CBR structures present in the argumentation of the parties, of the court of the 1st instance and in the final decision. The scheme makes it possible to infer arguments from the text of the case and to observe the mutual relations between arguments, including the attack relations and ordering of arguments.

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# *Reasoning by Cases in Structured Argumentation*

Mathieu Beirlaen

Jesse Heyninck

Christian Strasser

Ruhr University Bochum

mathieu.beirlaen@rub.de

jesse.hey ninck@rub.de

christian.strasser@rub.de

<http://homepages.ruhr-uni-bochum.de/defeasible-reasoning/index.html>

## **Abstract**

We extend (a fragment of) the *ASPIC*<sup>+</sup> framework for structured argumentation so as to allow for applications of the reasoning by cases inference scheme for defeasible arguments. Given an argument with conclusion ‘*A* or *B*’, an argument based on *A* with conclusion *C*, and an argument based on *B* with conclusion *C*, we allow the construction of an argument with conclusion *C*. We show how our framework leads to different results than other approaches in non-monotonic logic for dealing with disjunctive information, such as disjunctive default theory or approaches based on the OR-rule (which allows to derive a defeasible rule ‘If (*A* or *B*) then *C*’, given two defeasible rules ‘If *A* then *C*’ and ‘If *B* then *C*’). This way, we raise new questions regarding the subtleties of reasoning defeasibly with disjunctive information.

## **Extended abstract**

Formal argumentation has emerged as a promising approach to the modeling of defeasible reasoning. In [1], a set of arguments  $\mathcal{A}$  is arranged in a directed graph  $(\mathcal{A}, \text{Att})$  where  $\text{Att} \subseteq \mathcal{A} \times \mathcal{A}$  is a relation of argumentative attack. Given such a graph, argumentative semantics specify criteria for selecting acceptable sets of arguments, i.e. sets of arguments that form coherent and defensible positions in a debate. The field of structured argumentation has emerged in an attempt to give logical structure to both arguments and the attack relation.

We will present a structured argumentation framework based on (a fragment of) the *ASPIC*<sup>+</sup> framework [5] where arguments can be built using premises  $\Gamma$ , strict rules  $\mathcal{S}$  and defeasible rules  $\mathcal{D}$ . Defeasible rules are represented as  $A_1, \dots, A_n \Rightarrow \phi$  and are read intuitively as “If  $A_1, \dots, A_n$  are true then under normal circumstances,  $\phi$  is the case as well”. Strict rules are represented as  $A_1, \dots, A_n \rightarrow \phi$ . The following requirement ensures that the set of strict rules is closed under classical logic:  $A_1, \dots, A_n \rightarrow \phi \in \mathcal{S}$  iff  $\{A_1, \dots, A_n\} \vdash \phi$ . We allow a set of arguments  $\mathcal{A}(\Gamma, \mathcal{S}, \mathcal{D})$  to be recursively build up as follows:

- i.  $\phi \in \mathcal{A}(\Gamma, \mathcal{S}, \mathcal{D})$  if  $\phi \in \Gamma$ .  
 $\text{conc}(\phi) = \phi$ .
- ii.  $A_1, \dots, A_n \rightarrow \psi \in \mathcal{A}(\Gamma, \mathcal{S}, \mathcal{D})$  if  $A_1, \dots, A_n \in \mathcal{A}(\Gamma, \mathcal{S}, \mathcal{D})$  and  
 $\text{conc}(A_1), \dots, \text{conc}(A_n) \rightarrow \psi \in \mathcal{S}$ .  
 $\text{conc}(A_1, \dots, A_n \rightarrow \psi) = \psi$ .
- iii.  $A_1, \dots, A_n \Rightarrow \psi \in \mathcal{A}(\Gamma, \mathcal{S}, \mathcal{D})$  if  $A_1, \dots, A_n \in \mathcal{A}(\Gamma, \mathcal{S}, \mathcal{D})$  and  
 $\text{conc}(A_1), \dots, \text{conc}(A_n) \Rightarrow \psi \in \mathcal{D}$   
 $\text{conc}(A_1, \dots, A_n \Rightarrow \psi) = \psi$ .

In this contribution, we extend the *ASPIC*<sup>+</sup> framework with the following scheme for building arguments:

- iv.  $A_0, [A_1], [A_2] \rightsquigarrow \phi \in \mathcal{A}(\Gamma, \mathcal{S}, \mathcal{D})$  if  $\text{conc}(A_0) = \psi_1 \vee \psi_2, A_0 \in \mathcal{A}(\Gamma, \mathcal{S}, \mathcal{D}), A_1 \in \mathcal{A}(\Gamma \cup \{\psi_1\}, \mathcal{S}, \mathcal{D}), A_2 \in \mathcal{A}(\Gamma \cup \{\psi_2\}, \mathcal{S}, \mathcal{D})$  and  $\text{conc}(A_1) = \text{conc}(A_2) = \phi$ .  
 $\text{conc}(A_0, [A_1], [A_2] \rightsquigarrow \phi) = \phi$ .

**Example 1.** Given  $\Gamma = \{p \vee q\}, \mathcal{D} = \{p \Rightarrow u, u \Rightarrow s, q \Rightarrow t, t \Rightarrow s\}$  and  $\mathcal{S}$  as defined above, we have that  $p \vee q, [(p \Rightarrow u) \Rightarrow s], [(q \Rightarrow t) \Rightarrow s] \rightsquigarrow s \in \mathcal{A}(\Gamma, \mathcal{D}, \mathcal{S})$ .

We say that an argument  $A$  attacks an argument  $B$  if  $\text{conc}(A) \equiv \neg\phi$ , where  $\phi = \text{conc}(B)$ . Furthermore,  $A$  will also attack every argument that uses  $B$  as a sub-argument. Moreover, we define an attack relation for filtering out incoherent arguments, a well-known problem in *ASPIC*<sup>+</sup> [6].

Considering that there are arguments using the scheme *iv*, there can now be several paths to the argument's conclusion. For instance, there are two paths leading to the conclusion  $s$  in the argument from Example 1. One path runs via  $p$  and  $u$ , the other via  $q$  and  $t$ . When defining attacks for arguments constructed using scheme *iv*, one can take either a more cautious approach or a more credulous approach. In the cautious approach, all the paths to an argument's conclusion have to be safe, i.e. for an attack on an argument  $A, [B], [C] \rightsquigarrow \phi$  to succeed it suffices that (either a sub-argument of  $A$  is rebutted) or (a sub-argument of  $B$  is rebutted) or (a sub-argument of  $C$  is rebutted). In the credulous approach on the other hand, we require that (either a sub-argument of  $A$  is rebutted) or (a sub-argument of  $B$  is rebutted and a sub-argument of  $C$  is rebutted). In this approach, there are thus rather strong conditions on attackers of disjunctive arguments: for an attack to be valid it is required to show that all paths to the conclusion of the argument are disputable. We can model both alternatives via different attack definitions.

**Example 2.** Let  $\Gamma' = \Gamma \cup \{\neg t \wedge \neg u\}$ , with  $\mathcal{S}$  and  $\mathcal{D}$  as in Example 1. Based on the new premise set we can construct an argument with conclusion  $\neg t$  and an argument with conclusion  $\neg u$ . For the argument  $p \vee q, [(p \Rightarrow u) \Rightarrow s], [(q \Rightarrow t) \Rightarrow s] \rightsquigarrow s$ , the path via  $(p \Rightarrow u) \Rightarrow s$  is rebutted by the argument for  $\neg u$ , while the path via  $(q \Rightarrow t) \Rightarrow s$  is rebutted by the argument for  $\neg t$ . In the credulous approach we need both rebuttals in order to attack the argument  $p \vee q, [(p \Rightarrow u) \Rightarrow s], [(q \Rightarrow t) \Rightarrow s] \rightsquigarrow s$ . In the cautious approach each of these rebuttals is by itself sufficient to attack the larger argument. In other words, in the cautious approach it would be sufficient to have an argument for  $\neg t$  or an argument for  $\neg u$  in order to attack the argument  $p \vee q, [(p \Rightarrow u) \Rightarrow s], [(q \Rightarrow t) \Rightarrow s] \rightsquigarrow s$ .

Various acceptability semantics have been defined for selecting coherent subsets of arguments representing rational stances in a debate (grounded, preferred, stable, etc.). All of these are readily applicable in our framework.

We will compare our system with two other approaches for dealing with disjunctive information in the context of defensible implications. In particular, we will point to differences with both disjunctive default logic [2] and approaches based on the OR-rule, as used in e.g. system **P** [3] and in several Input/Output logics [4]. The OR-rule allows to derive a defeasible rule 'If ( $A$  or  $B$ ) then  $C$ ', given two defeasible rules 'If  $A$  then  $C$ ' and 'If  $B$  then  $C$ '. By showing these differences, we raise new questions regarding the subtleties of reasoning defeasibly with disjunctive information.

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# Normal Default Rules as Epistemic Actions

Michal Dančák  
Michal Peliš  
Department of Logic  
Faculty of Arts  
Charles University in Prague  
mdancak@seznam.cz  
michal.pelis@ff.cuni.cz

## Abstract

We would like to introduce the idea of ‘translating’ *normal default rules* in a slightly rearranged *action model logic*. So, we try to provide a dynamic epistemic background for *normal default logic* with operational semantics [1]. In this paper, we present the basic idea and definitions using the single-agent version of action model logic. In the talk, we move on to problems of the multi-agent version.

**Keywords:** action model logic, epistemic logic, normal default logic

## 1. Common reasoning with default rules

Using the word *reasoning* we mostly mean ‘private’ act of a subject.<sup>1</sup> Common reasoning that we do all the time is often based on two kinds of information. The first one, *hard information*, is information that we are obliged to accept; we are sure of it, it is our knowledge and trusted data. The other one, *soft information*, is something that ‘typically happens’, it is very likely that things go that way. Reasoning based exclusively on hard information would be ideally deductive. However, it is not possible in common reasoning. There are many typical situations that are produced by our experience. Some of them are in contradiction, some of them are incomplete. Nonetheless, we have to do conclusions even if there is a lack of hard information. Such conclusions can be out of the scope of classical (deductive) consequence relations.

Let us imagine that we know that Anne is a student of a faculty of arts. A typical student from a faculty of arts does not like mathematics and we could conclude by default that Anne does not like mathematics. The knowledge that Anne attends a faculty of arts together with our prejudice that students from this faculty do not like math can form a (default) ‘rule’:

If  $AnneStudOfArtFac$ , then  $\neg AnneLikeMath$  under condition that the conclusion  $\neg AnneLikeMath$  is not in a conflict with our current knowledge.

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<sup>1</sup>Let us call a ‘reasoning subject’ an *agent* as this term is used in epistemic logic. The word ‘private’ will be discussed later on.

Later we obtain an information that Anne studies logic. Well, if she studies logic, she might like mathematics. Similarly with this information, we can formulate a ‘rule’:

If *AnneStudOfLog*, then *AnneLikeMath* under condition that the conclusion *AnneLikeMath* is not in a conflict with our current knowledge.

More formally, if we put hard information together, we obtain a set (of facts)

$$\Gamma = \{ \text{AnneStudOfArtFac}, \text{AnneStudOfLog} \}$$

and our reasoning about Anne’s liking for math can follow rules like these:<sup>2</sup>

$$\frac{\text{AnneStudOfArtFac} : \neg \text{AnneLikeMath}}{\neg \text{AnneLikeMath}}, \frac{\text{AnneStudOfLog} : \text{AnneLikeMath}}{\text{AnneLikeMath}}$$

Note that once we conclude that Anne does not like mathematics we may not use the second rule because its presupposition is in conflict with our current knowledge, i.e., with the conclusion that Anne does not like math.<sup>3</sup>

## 2. Normal defaults as actions

In *action model logic* we work with the idea that there is a finite group of agents and these agents change their epistemic states with respect to ‘new’ information produced by verbal as well as non-verbal actions [2]. Reasoning of an agent *i* about ‘Anne’s liking for math’ can be such action. We called this reasoning ‘private’ because it mostly happens as a change of *i*’s epistemic state without any communication with other agents. Nonetheless, it need not be completely private, other agents can know both hard and soft information that are available to *i*, and can follow all *i*’s steps in reasoning.

To incorporate default reasoning inside action model logic we have to understand default rules as actions. More formally, a normal default rule  $\frac{\varphi:\psi}{\psi}$  changes agent’s epistemic model such that it separates  $\psi$ -worlds (possible worlds where the formula  $\psi$  is true) from  $\neg\psi$ -worlds. This forms a base for a possible application of other defaults. From now on, the agent is ready to work with preferred (*designated*)  $\psi$ -worlds. If the agent *i* goes on with some other default, she only checks the validity of a (new) prerequisite with respect to the possible worlds where  $\psi$  is true. It does not mean that  $\neg\psi$ -worlds are deleted from the model—they are now distinguishable for *i*.

This idea brings us to a small modification of epistemic models and actions from [2, Chapter 6]:

**Definition 1.** A (default) epistemic model *M* is an S5-structure  $(W, R_i, V, X_i)$  where *W* is a non-empty set of possible worlds,  $R_i \subseteq W^2$  is an accessibility relation of an agent *i*, *V* is a valuation function (from the set of atomic formulas to *W*), and  $X_i \subseteq W$  is a non-empty set of designated possible worlds for an agent *i* such that whenever  $uR_iv$ , it holds that  $u \in X_i$  iff  $v \in X_i$ .<sup>4</sup>

In the agent’s reasoning (esp. in the application of a default rule) the agent *i* ‘ignores’ all the states outside of  $X_i$ . The designated worlds are not connected via  $R_i$  to those that are not designated.

<sup>2</sup>The rule is in the form of a fraction  $\frac{\varphi:\psi}{\psi}$ ; prerequisite ( $\varphi$ ) is written before the colon together with what is presupposed ( $\psi$ ) behind the colon, a consequent ( $\psi$ ) is under the line. Rules, where the presupposition is the same as the consequent, are called *normal*.

<sup>3</sup>And vice versa. There are two extensions of our hard information  $\Gamma$ .

<sup>4</sup>The satisfaction relation  $\models$  between possible worlds and formulas is defined in the standard way.

**Definition 2.** A normal default rule  $\frac{\varphi:\psi}{\psi}$  is epistemically applicable by an agent  $i$  in an epistemic model  $M$  iff for each  $w \in X_i$ :  $(M, w) \Vdash K_i\varphi$  and  $(M, w) \Vdash \hat{K}_i\psi$  and  $(M, w) \Vdash \hat{K}_i\neg\psi$ .

An action  $(\varphi : \psi/\psi)^i$  corresponding to a normal default  $\frac{\varphi:\psi}{\psi}$  (used by an agent  $i$ ) will be understood as a two-node action model  $\mathbf{D}^i = (\mathbf{W}, \mathbf{R}_i, \text{pre}, \mathbf{X}_i)$ <sup>5</sup> where

- $\mathbf{W} = \{s, t\}$
- $(s, t) \notin \mathbf{R}_i$ , but  $(s, s) \in \mathbf{R}_i$  and  $(t, t) \in \mathbf{R}_i$
- $\text{pre}(s) = \psi$  and  $\text{pre}(t) = \neg\psi$
- $\mathbf{X}_i = \{s\}$

The action  $\mathbf{D}^i$  updates (changes) an epistemic model  $M = (W, R_i, V, X_i)$  into a (new) epistemic model  $M'$  if the corresponding default rule is applicable (Definition 2).<sup>6</sup> The resulting (updated) epistemic model  $M' = (M \otimes \mathbf{D}^i)$  is a structure  $(W', R'_i, V', X'_i)$  where

- $W' = \{(w, s) \mid w \in W \ \& \ s \in \mathbf{W} \ \& \ (M, w) \Vdash \text{pre}(s)\}$
- $(w, s)R'_i(w', s')$  iff  $(wR_iw' \ \& \ sR_i s')$ , for  $w, w' \in W$  and  $s, s' \in \mathbf{W}$
- $(w, s) \in V'(p)$  iff  $w \in V(p)$ , for  $(w, s) \in W'$  and atomic formula  $p$
- $(w, x) \in X'_i$  iff  $w \in X_i \ \& \ x \in \mathbf{X}_i$ , for  $w \in W$  and  $x \in \mathbf{W}$

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<sup>5</sup>An action model is very similar to an epistemic model. The difference lies in precondition function  $\text{pre}$  from  $\mathbf{W}$  to the set of all formulas.

<sup>6</sup>The applicability of a default takes over the role of precondition in standard action model logic.

# *Slippery Slopes and other Consequences*

Martin Hinton  
University of Łódź  
Department of Legal Theory  
mdhinton@tlen.pl  
<http://filologia.uni.lodz.pl/hinton/>

## **Abstract**

This paper provides a critique of Douglas Walton's understanding of Slippery Slope Arguments and the argument scheme based upon it. It is argued that he fails to discern the important difference between real-world bad consequences and the argumentational consequences which are usually implied in such arguments. This is followed by the development of a more accurate characterisation and a simple scheme with critical questions. The latter part of the paper considers other arguments from consequences and compares how the 'gravity' of those arguments differs from that of the slippery slope.

**Keywords:** Douglas Walton, Slippery Slope Arguments, Gravity of Argumentation

## **Extended abstract**

Douglas Walton (2015) recently outlined his proposal for an argumentation scheme for slippery slope arguments (SSAs). Walton considers the proposed characterisations of SSAs offered by other authors and finds them incomplete or otherwise unsatisfactory. He bases his argumentation scheme on a 10 part analysis of the nature of such arguments and illustrates these stages of the argumentation process with examples, in particular focussing on the case of drug addiction as a clear slippery slope which it is reasonable to warn against. Walton's characterisation of SSAs is most notable for its introduction of the notion of a 'gray area' at some undefined point of the slope where control is lost and further descent becomes unstoppable.

The first aim of this paper is to illustrate where previous attempts at SSA characterisation have gone wrong, and to provide an analysis which better captures their true nature. The first part describes Walton's arguments in support of his views on SSAs and also considers the characterisations put forward by other researchers. All of these are found wanting due to their failure to capture the essence of the slippery slope and their inability to distinguish SSAs from other consequentialist forms of argument. Walton, it is argued, is particularly guilty of this, and makes no attempt to respond to criticism of his earlier work which points out his confusion between chains of reasoning and chains of events (Van Der Burg, 1993). I also argue that Walton's choice of examples is unfortunate: while drug addiction is certainly a slippery slope for the addict, it does not represent a slippery slope of argumentation, since the choice of the addict to continue to abuse the substance is not taken after reasonable reflection upon the arguments.

The notion of the 'grey area' is also questioned, since the metaphor of the slope suggests that once the foot is placed upon the slippery decline, control is already lost: there is no reason to think that slippery slopes become slippery half-way down.

The second part of the paper puts forward a clearer analysis of what is special about SSAs and allows them to be easily distinguished from their close cousins: arguments from precedent, arguments from consequences, and arguments from consistency. It is argued that all SSAs, properly so-called, claim that reaching a certain conclusion, A, involves the negation of a principle, P, and that that principle is necessary to argue against further conclusions (B,C...) which are considered unacceptable. There are, therefore, three ways in which an SSA can go wrong: 1. That the principle P does not need to be broken to reach A. 2. That the principle P is not necessary to argue successfully against B,C... 3. That conclusions B,C... are not, in fact undesirable. This characterisation is tested against a number of examples and some possible objections are considered and rejected.

This analysis of SSAs leads to the development of a concept of 'the gravity of argumentation': in this case the force which pulls us down the slope towards the undesirable consequences. It is argued that a distinction can be made between different arguments from consequences on the basis of what type of 'gravity' the argument is subject to. Thus it is seen that, although they are not structurally identical, arguments from consistency are driven forward by the same, logical, force as SSAs, where straightforward arguments from bad consequences take their strength from purely moral considerations, backed up by empirical evidence. Arguments from precedent, however, require both a logical and a moral force, in that they generally rely on both the need for logical consistency and a sense of fairness or equality.

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# *A formal model of an argumentative dialogue in the management of emotions\**

Magdalena Kacprzak<sup>1</sup>

Krzysztof Rzeńca<sup>2</sup>

Anna Sawicka<sup>3</sup>

Andrzej Zbrzezny<sup>4</sup>

Katarzyna Żukowska<sup>2</sup>

<sup>1</sup>Faculty of Computer Science

Białystok University of Technology

m.kacprzak@pb.edu.pl

<sup>2</sup> Faculty of Psychology

SWPS University of Social Sciences and Humanities in Warsaw

krzenca@swps.edu.pl

kzukowska@swps.edu.pl

<sup>3</sup> Polish-Japanese Academy of Information Technology in Warsaw

asawicka@pjwstk.edu.pl

<sup>4</sup> Faculty of Mathematics and Natural Science

Jan Długosz University in Częstochowa

a.zbrzezny@gmail.com

## **Abstract**

In our research we focus on designing an interactive tool which can support recognition of human emotions that occur in dialogues. It will be used as an aid in learning how to manage emotions during argumentative dialogues. For this purpose we analyze human dialogues and build a system to apply argumentative dialogues in human-computer communication. In this way, we gather a collection of examples illustrating the typical human's reactions and use them to explain mechanisms that appear in dialogues. Our inspirations are dialogue games. In this work we present a theoretical background of the project, i.e., we provide a formal system to represent argumentative dialogue protocols.

**Keywords:** argumentative dialogue, emotions, protocol, formal model

## **1. Management of emotions in dialogue**

The aim of our research is to create a formal model which will allow interlocutors to improve communication by interpersonal skills coaching. The studies focus on argumentative di-

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alogues. As a case study we want to look closer at the dialogues between parents (teachers) and children. In these dialogues, parents learn how to manage emotions to achieve the intended behavior, e.g., the child does homework, tidies the room, doesn't use offensive language, etc. Every statement (of a child, but also an adult) contains content (usually formulated explicitly) and emotions (often hidden). It can happen that we understand the content but we are not able to properly interpret the accompanying emotions. We should also be aware how our own emotions are perceived by the interlocutor. Understanding our emotions and interlocutor's emotions especially in difficult situations enables effective communication. Emotions are important guideposts, which allow us to decide how to handle a conversation [2]. The main task of the project is to show that through recognizing child's emotions we are teaching him to identify them. Naming emotions and being aware of them help to focus on finding a solution to the problem. In our study we are showing what emotions accompany given statements and how the way we talk can affect their intensity. This mechanism is shown using five emotions: fear, disgust, joy, sadness, and anger [6].

In our research we design and implement a framework for communication between a user and a machine which allows us to better understand emotions that appear during human dialogues. We plan to create a tool that will support the personal development associated with improvement of emotions management, i.e., identifying and naming them. This is particularly important for training teachers, educators, psychologists, and parents. This process can take place between a human, which plays a role of a student, and a software agent, which plays a role of a teacher. Many parents and teachers want to show their support to the children, but they do not know how. Instead, they unwittingly use methods that produce the opposite effect. Parents are not lacking motivation, but do not have the appropriate skills. Our tool can be helpful with this matter.

## 2. Model for parent-child argumentative dialogue

In this work we propose a formal framework for dialogue and, more specifically, we define the dialogue protocol modelling conversation between a parent and a child. In such a dialogue the parent tries to convince the child by giving various arguments. These arguments are not necessarily justified rationally, e.g., due to having lunch you will be healthy, but they rather teach the child how to recognize its own emotions and manage them.

We start out by defining a mathematical model for argumentation dialogue game, which uses the concept of interpreted systems and Kripke structures. In the model, we assume that the set of players of a dialogue game consists of two players: Parent ( $P$ ) and Child ( $H$ ),  $Pl = \{P, H\}$ . To each player  $p \in Pl$ , we assign a set of possible actions marked with  $Act_p$  and a set of possible local states marked with  $L_p$ . An action from  $Act_p$  can influence participant's commitments and emotions. It represents locution performed by a player and the content of this locution. Results of actions are determined by *evolution function* (for details see [4]). Player's local state  $l_p \in L_p$  consists of the player's *commitments*, *emotions*, and *goals*,  $l_p = (C_p, E_p, G_p)$ . Player's commitments and goals are elements of a fixed topic language, which allows expressing the content of locutions. Thus,  $C_p$  and  $G_p$  are sets of such expressions. These sets may be subject to change after a player's action. More specifically, the player can add or delete the selected expression. Emotions which we consider are: fear, disgust, joy, sadness, and anger. Their strength (intensity) is represented by natural numbers from the set  $\{1, 2, \dots, 10\}$ . Thus,  $E_p$  is a 5-tuple consisting of 5 values, which may also change after a certain action. It is worth highlighting here that a change in the intensity of the emotions is dependent on the type of locution and, perhaps even more, on its content. A global state  $g \in G$ , where  $G$  is the set of all global states, is a triple consisting

of a history of a dialogue (formally it is a sequence of moves)  $d(g)$  and players' local states  $l_P(g)$  and  $l_H(g)$  corresponding to a snapshot of the system at a given time:  $g = (d(g), l_P(g), l_H(g))$ .

An *interpreted system* for a dialogue game is a tuple  $IS = (I, \{L_p, Act_p\}_{p \in Pl})$ , where  $I \subseteq G$  is the set of initial global states. The actions executed by players are selected according to a *protocol function*, which maps a global state  $g$  to the set of possible *global actions* (for details see [3]). Interpreted systems are traditionally used to give semantics to an epistemic language enriched with temporal connectives based on linear time. As our basic temporal language we use the logic CTL by Emerson and Clarke, which is commonly used to verify the dialogues in artificial intelligence systems [1] and extend it by adding commitments, emotions, goals, dynamic and past components.

It is a common knowledge that emotions are important in contacts people-to-people. Some of the artificial intelligence systems are also equipped with modules which allow for exploring the role of emotions in e.g. decision making (see [5]). We use our model to show that very important, if not the most important thing in the successful argumentation carried out between parent and child is the ability to manage emotions. To achieve this we need to introduce a system for argument evaluation, which depends on interlocutor's attitude. In this research we propose the one similar to the methodology for persuading people through argumentative dialogues proposed by Rosenfeld and Kraus [7]. In the talk we will show more details of our approach and discuss the difficulties that we faced when designing the model for parent-child dialogue.

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# *Erotetic Reasoning Corpus. A data set for research on natural question processing*<sup>\*</sup>

Paweł Łupkowski  
Mariusz Urbański  
Adam Mickiewicz University in Poznań  
Pawel.Lupkowski@amu.edu.pl  
Mariusz.Urbanski@amu.edu.pl  
<http://reasoning.edu.pl>

## **Abstract**

The aim of this talk is to present the Erotetic Reasoning Corpus which constitutes a data set for research on natural question processing. We describe the theoretical background, language data and tags used for the annotation process. We also present elements of the ERC interface: search engine and XML/L<sup>A</sup>T<sub>E</sub>X parser. We also discuss ERC-based study and the potential areas of use of ERC.

**Keywords:** questions, logic of questions, question processing, erotetic reasoning

## **1. Motivations**

The Erotetic Reasoning Corpus (ERC) is a data set for research on natural question processing. Intuitively, we are dealing with question processing in a situation when a question is not followed by an answer but with a new question or a strategy of reducing it into auxiliary questions. Usually, such a situation takes place when an agent wants to solve a certain problem (expressed in a form of an initial question) but is not able to reach the solution using his/her own information resources. Thus new data, collected *via* questioning is necessary. This phenomenon is studied within such theoretical frameworks as Inferential Erotetic Logic (IEL) [11], [12], [6]; inquisitive semantics [4]; or KoS [2]. Natural question processing constitutes also an attractive subject for empirical research. ERC aims at facilitating such a research concerning question processing in natural language.

Our logical framework of choice is IEL. This logic focuses on inferences whose premises and/or conclusion are questions (erotetic inferences). IEL offers some straightforward tools for modelling erotetic inferences. What is especially important from our perspective is that IEL not only gives semantical analysis of erotetic inferences but also proposes certain criteria of their validity (the most essential notion in our case is that of erotetic implication; canonical [12], falsificationist [3] and weak one [9]).

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## 2. Architecture

The whole ERC consists of 552 files (212.685 words). All the data is in Polish, however the tag-set used for the annotation allows for the data analysis for English-speaking researchers. The corpus consists of the language data collected in the previous studies on the question processing phenomenon. These are: Erotetic Reasoning Test, QuestGen and Mind Maze.

*Erotetic Reasoning Test* [9] contains 3 items. Each item consists of a detective-like story in which the initial problem and evidence gained are indicated. The task is to pick a question (one out of four), each answer to which will lead to some solution to the initial problem. The subjects are asked to justify their choices.

*QuestGen* [5], [7] is an on-line game in which, two randomly chosen players are engaged in solving a detective puzzle. One of them plays as the Detective, while the other is called the Informer. The aim for the Detective is to solve the presented puzzle by questioning the Informer.

*Mind Maze* is a card game. In the game the game master (GM) tells a short story (inspired by true events) and the objective of the player is to figure out how the story happened by asking questions to GM. Mind Maze was used as the core element for the semi-structured study of question processing (see [10]). The researcher played the role of the GM and subjects were players. Game sessions were recorded and then transcribed.

## 3. Tagging

XML is used to annotate the ERC data. Tagging schema for the ERC has three layers:

1. *Structural*—representing the structure of tasks used for the studies described in Section 2. Here we distinguish elements like: instructions, justifications, different types of questions and declaratives.
2. *Inferential*—which allows for recognizing normative erotetic concepts.
3. *Pragmatic*—representing various events that may occur in the dialogue, like e.g. long pauses. It also contains tags that allow expression of certain events related to the types of tasks used (like e.g. when forbidden question is used).

## 4. ERC interface

**Search engine.** The ERC search engine provides the following functionalities: displaying the XML files (with syntax highlighting), searching for words, searching using regular expressions, searching for particular ERC tags.

**XML/L<sup>A</sup>T<sub>E</sub>X parser** [1]. The tool allows for extracting a solution of a given type and typeset it in a dialogical form or extracting and preparing a graphical representation of a given entry structure (in a tree-like form). For this purpose the structural layer of annotation is used. The parser will be especially useful for preparing papers and presentations based on the ERC data. Thus the choice of using L<sup>A</sup>T<sub>E</sub>X as the output format for our tool.

## 5. Applications

In our opinion the ERC's potential scope of use is wide and reaches far beyond studies of the normative logical concepts vs. real erotetic reasonings. The potential applications may cover the following exemplary areas of interests:

- Linguistic studies of the way questions are formulated for different contexts.
- Research on dialogue management.
- Problem solving studies concerning strategies of handling with question decomposition.
- Studies focused on the way the question should be asked (or an initial problem/task should be formulated) in order to make the solution easier to reach.

In our talk we will present a study concerning the last domain. We will address a question, whether deductive stories designed accordingly to the erotetic logic tools are easier (in terms of time and correctness) to solve for the QuestGen players, and how an answer to this question might be potentially useful for a game design purposes.

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# *Intuitive Explanations*\*

Jerzy Pogonowski  
Department of Logic and Cognitive Science  
Adam Mickiewicz University, Poznań  
pogon@amu.edu.pl

## **Extended abstract**

We are going to discuss the role of intuitive explanations in mathematical works. Argumentation involving intuitive ideas is of course ubiquitous in teaching of mathematics. One can also, though rather scarcely, find references to intuitive views in original research papers of professional mathematicians. Here we limit ourselves mostly to the former case. Our sources are a few chosen textbooks of advanced parts of mathematics published in the last decades.

The use of intuitive explanations in textbooks is a mere fact. One can ask several questions concerning this fact:

1. What is the goal of such explanations?
2. What are the effects of them?
3. What are the intuitive explanations based on?
4. What are the connections between mathematical intuitions (of professional mathematicians) responsible for mathematical discoveries and the intuitive explanations used in the textbooks in order to promote understanding of the already established mathematical ideas?

We will modestly try to provide tentative answers to these questions in our talk. Quite obviously, the goal of intuitive explanations is to evoke a better understanding of the discussed mathematical ideas. But how can we be sure that such-and-such explanation appropriately serves this goal? Plain translation of a symbolic formula into English should not be considered as a complete explanation. However, there should exist a strong preservation of meaning between the explained idea and its explanation. One might thus suppose that this procedure is based on analogy or metaphor. But notice that the idea which we want to explain intuitively is more complicated than the offered explanation: intuitive explanations are, as a rule, certain simplifications (in a sense, they are *converse metaphors*).

The literature reports on several methods or even tricks used in order to make sophisticated ideas more accessible to the students, e.g. to familiarize them with the concepts of infinity, continuity, derivative, integral, some further advanced arithmetical, algebraic or topological notions.

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The efficiency of such methods can be measured by the ability of students to solve problems (of a certain kind) by themselves. We think that at least two issues are interesting in this respect. First, has a given method a local character only or can it be applied to the situation where the student meets a completely new kind of problem, not explained intuitively before? For example, can she apply a kinematic explanation of derivative to, say, variational problems? Second, what are historical changes in the style of intuitive explanations? For example, does students' familiarity with computers (computer programs) influence the style of modern explanations?

We meet several types of intuitive explanations in mathematical textbooks. They are referring, for example, to:

1. *Language*. Some authors limit themselves to mere translations of symbolic notation into natural language expressions. Verbalization of a mathematical idea may be fruitful but it does not guarantee better understanding.
2. *Common sense*. References to everyday experience should be easily understood but they should be used carefully. Due to, among others, several cognitive biases common sense reasoning may diverge dramatically from mathematical inferences.
3. *Vision*. This includes drawings, pictures, diagrams and the like. Their role is supplementary and they should not be too suggestive.
4. *Physics*. The inspirations for creation of large parts of mathematics come from investigations in physics. It is thus natural to explain mathematical ideas going back to their physical roots.
5. *Human behavior*. Funny enough, on average people think that they better understand their behavior than mathematical concepts.
6. *Cross-domain references in mathematics itself*. Such explanations are, in our opinion at least, most interesting. Applying algebraic tools in topology enabled us to solve difficult topological problems via transformation into (as a rule, much simpler) algebraic problems. Some facts from the theory of functions of a real variable can be better understood when viewed from the perspective of complex analysis. These facts may suggest that we could successfully explain ideas from one mathematical domain by reference to other domain, presumably better understood by the students themselves.

We will provide examples of these types of explanations in the talk. It may be added that we are interested in "serious" explanations only, thus omitting such cases where an intuitive digression is made for stylistic purposes only.

The last question mentioned at the beginning of this abstract has at least two aspects. First, it concerns ties between *invention* and *comprehension* of mathematical ideas. It seems that in order to grasp the meaning of a mathematical concept it is not necessary to follow exactly the way of thinking of its inventor. Moreover, due to the currently accepted style of writing, mathematical texts do not reveal the context of discovery of the results contained in them. Second, there may exist much easier (from a didactic point of view) ways of discussing the intended meaning of a particular concept than the way on which it has been originally introduced by its inventor. A mathematical concept once brought into legitimate being in the realm of mathematics (i.e. accepted by the mathematical community) interacts with other concepts, may be situated in new contexts, may generate a bunch of new facts, etc. All this may broaden the possibilities of its intuitive explanations.

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*Based on what should we formulate a legitimate definition of quasi-logical arguments?  
Case study of arguments of reciprocity*

Iva Svačinová  
Department of Philosophy  
Faculty of Arts  
Masaryk University  
iva.svacinova@mail.muni.cz

**Abstract**

The paper attempts to formulate an explicit definition of the so-called quasi-logical arguments, the phenomenon originally defined by Perelman and Olbrechts-Tyteca in the new rhetoric. The new rhetoric is widely criticized as a model that provides a vague definition of its categories leading to possible different understanding categories and subsequently to different results when the categories are used in the analysis. The paper offers a case study of a particular subtype of quasi-logical arguments, so-called arguments of reciprocity.

The paper is divided into two parts. The first part analyzes the sources of vagueness allowing different interpretations. The difference of interpretations is demonstrated by the analysis of three recent attempts to define arguments of reciprocity explicitly. The difference of interpretations raises the question of legitimacy of these interpretations given to the elements of the original category of Perelman and Olbrechts-Tyteca. It is desirable to provide such interpretation, which corresponds to phenomena that were considered as falling under the original category. The second part is dedicated to the suggestion of definitional approach, which leads to a legitimate definition. As a suitable definitional approach seems to be an analysis of argumentative fragments from that Perelman and Olbrechts-Tyteca proceeded in the formulation of their categories. In this paper, for the analysis of fragments is chosen the pragma-dialectical model of argumentation, which allows conceiving argumentative techniques functionally, in respect to the solution of conflict of opinion, and explores rhetorical strategies implemented by these techniques. The paper presents own definition of the phenomena falling under arguments of reciprocity. It grasps them as specific structural variations of argument schemes in the pragma-dialectical sense.

**Keywords:** definition, new rhetoric, pragma-dialectics, quasi-logical arguments

## Extended abstract

Quasi-logical arguments are originally defined in the new rhetoric of Chaim Perelman and Lucie Olbrechts-Tyteca [4]. According to the authors, they are type of argument schemes that are implemented through structural and lexical assimilation of arguments to mathematical or logical demonstrations. In the new rhetoric, with the quasi-logical arguments is connected claim that is in the paper called “thesis about the effectiveness of quasi-logical arguments”: Perelman and Olbrechts-Tyteca argue that the use of a quasi-logical form of argumentation is an effective strategy for persuading an audience [4, p. 193]. According to them, people tend to accept arguments that resemble formal demonstrations because formal demonstrations enjoy high prestige in our culture.

The new rhetoric is generally appreciated as an inventory of original rhetorical techniques, according to Johnstone “there are myriads of rhetorical techniques that they seem to have noticed for the first time” [3, p. 102]. Simultaneously is questioned the applicability of new rhetorical typology in analysis, because the definitions of phenomena falling under individual categories are too vague and enable numerous interpretations (cf. [5, p. 122]). It is difficult to decide which of these interpretations legitimately captures the phenomena that were originally considered by Perelman and Olbrechts-Tyteca. The problem of demarcation of elements falling under the vaguely defined categories in the new rhetoric is demonstrated in this paper in a case study of a particular subtype of quasi-logical arguments: arguments of reciprocity.

In the first part of the paper are identified three sources that together enable a final difference of interpretations of categories: (1) absence of a general definition of the concept of argument scheme in the new rhetoric, which allows to interpreters to have different initial ideas of what can be regarded as argument scheme; (2) complex structured definition of the categories in the new rhetoric, which enables to interpreters to construct a definition based on different levels of abstraction; (3) absence of a unequivocal demarcation of permissible meanings of terms on different levels of the definition, which enables to interpreters to determine permissible interpretations based on their own preferences.

Focusing specifically on the arguments of reciprocity, difference of their final interpretations lies in the fact that (1) Perelman and Olbrechts-Tyteca describe argumentative schemes as a type of *discursive techniques* or *loci*, which is very vague description without further specification. (2) The authors offer a multi-level definition of new rhetorical categories reflecting the inductive building definition by gradual abstractions from the empirical material. Arguments of reciprocity are specifically at the highest level of abstraction defined on the basis of their similarity with the formal logical principle of symmetry:

$$\text{Sym}(\mathbf{R}) =_{df} \forall xy (\mathbf{R}(x, y) \rightarrow \mathbf{R}(y, x)).$$

On the lower level of abstraction, they are defined as arguments, in which the components implement specific relations (“antecedent-consequent”, “transposition of points of view”, “opposites”, “inversion”, etc.). At the lowest level, it is a list of examples of the use of this argument scheme. (3) The authors do not offer at these levels of abstraction explicit definitions that clearly define the permissible interpretations. On the highest level of abstraction, the main problem lies in definition via similarity with the formal principle. It is not clearly defined, which variations of symmetrical relationship are relevant, what informal interpretations of the formal elements are permissible, how can be this principle used argumentatively. On the lower level is not defined sufficiently clearly what types of semantic relationships fall under different specifications. At the lowest level, we can find problematic a choice and form of empirical examples that are taken out from their original context and their analysis is not offered.

The combination of vague elements enables the formulation of many different interpretations. This fact is presented by analysis of three recent attempts to put forward a clear definition of arguments of reciprocity that have been currently provided by B. Garssen [1], B. Warnick and S. Kline [6] and F. Grasso [2]. It is shown how are their interpretation influenced by the initial understanding argumentation scheme, choice of level of abstraction, and specific choice of permissible meanings of concepts. The analysis systematically justifies differences of their interpretations.

The possibility of the formulation of many of various interpretations of one category raises the question of their legitimacy given to the original concept of Perelman and Olbrechts-Tyteca: which of them really grasps phenomena that were considered by authors? Identification of legitimate interpretation is crucial due to the thesis about the effectiveness of quasi-logical arguments. New rhetorical categories serve not only to describe the argumentative discourse, but they allow assessing the effectiveness of argument strategies. Specifically, identification of the use of the scheme of arguments of reciprocity in argumentative discourse allows us to assess the effectiveness of this strategy through their quasi-logical nature.

Category of arguments of reciprocity was formulated by Perelman and Olbrechts-Tyteca by gradual abstraction of some features of a sample of argumentative fragments, which the authors regarded as successful and readily recognized by audiences. However, it is entirely conceivable that the interpretations of the scheme resulting from contingent interpretative choices do not correspond to any of the techniques implemented in these fragments, may even be the cases of completely ineffective strategies that are not recognized by audiences. It is, therefore, desirable to find an interpretation corresponding techniques implemented in the sample of fragments.

The way, that seems to be promising, is to establish a definition of phenomena falling into the category of arguments of reciprocity based on the analysis of the lowest level: examples that are presented by Perelman and Olbrechts-Tyteca. The argumentative techniques identified through analysis of these fragments correspond certainly to elements of the original category because the original category was constructed on their basis. Perelman and Olbrechts-Tyteca confirm that "the examples we shall give here are analyzed as quasi-logical because this aspect of them is readily recognized." [4, p. 194] This approach leads to grasping only a part of the techniques that probably fall under the category of arguments of reciprocity, however, we can be certain regarding the legitimacy of our definition.

The definitional approach here proposed differs from "interpretive" approaches. The definition is based on analysis of individual fragments. The analysis can be principally carried out using different theoretical tools. In this paper is the pragma-dialectical argumentation model specifically chosen which allows functional description of these techniques due to the solution of conflict of opinion. The pragma-dialectical model extended by the concept of strategic maneuvering allows the exploration of the rhetorical effectiveness of these techniques (van Eemeren 2010).

The paper presents the results of analyses of these examples. It is shown that the arguments of reciprocity in this sense are not a single argument technique, but they are the sum of the techniques associated exclusively with argumentation stage of dialogue. They may be identified with some structural variations of the three pragma-dialectical argument schemes (analogical, symptomatic, causal), for which is characteristic (but not without exceptions) that they support a prescriptive standpoint, which may be formalized as " $R(x, y)$  should be implemented/evaluated as  $Y$ ". The type of reason and implemented argument schemes diverge. A complete overview of variations of argument schemes implemented in fragments is offered in the paper. In the conclusion is outlined the research of the rhetorical effectiveness of these variations via the concept of strategic maneuvering, especially the choice from potential topical.

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# *A Critical Discussion Game for Prohibiting Fallacious Arguments\**

Jacky Visser<sup>1</sup>

Katarzyna Budzyńska<sup>1,2</sup>

Chris Reed<sup>1</sup>

<sup>1</sup>Centre for Argument Technology

University of Dundee

j.visser@dundee.ac.uk

<http://arg-tech.org>

<sup>2</sup> Institute of Philosophy and Sociology

Polish Academy of Sciences

## **Abstract**

Fallacies are interpreted in the pragma-dialectical theory of argumentation as violations of the rules of critical discussion. Critical discussion is a normative model of ideal argumentative practice. Because the model combines a dialectical perspective on argumentation with a speech act perspective on communication, it is a good starting point for the computational modelling of argumentative dialogue and evaluation of fallacies. We formalise critical discussion as a dialogue game and implement it in the computational Dialogue Game Description Language. This implementation can subsequently be executed in Arvina, an online user interface. With Arvina, users can play out a game of critical discussion, together with other users or with artificial agents. In playing the game, Arvina will interact with and extend the AIFdb online repository of structured argumentation. Because the critical discussion game is based on the normative, pragma-dialectical model, the occurrence of fallacies is prohibited.

**Keywords:** critical discussion, Dialogue Game Description Language, fallacies

## **1. Extended abstract**

Fallacies are commonly interpreted in dialectical theories of argumentation as unreasonable ways of discussing. Within the pragma-dialectical theory of argumentation [4] — one of the most influential approaches in the modern field of argumentation studies — the model of a *critical discussion* is proposed as an ideal procedure for argumentative conduct. The rules that define the ideal model are intended to be instrumental for the reasonable resolution of differences of opinion, by regulating the means of defending and attacking standpoint and arguments.

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Fallacies are then conceived of as violations of the rules that constitute the model of a critical discussion. In other words, fallacies are those discussion moves that impede the reasonable resolution of a difference of opinion.

The normativity of the model is based on an implementation of a critical conception of reasonableness as opposed to a geometrical or an anthropological conception. The normative bite of the model makes it possible to employ it as a critical tool in the evaluation of argumentation. Because the model takes the full discussion into account, it can be used to identify not only inferential fallacies, but also fallacious discussion moves that obstruct the discussion in other ways. One advantage of this approach is that traditionally recognised fallacies such as *ad hominem* and *ad baculum* can be explained in a unified way as violations of the rules that define a reasonable argumentative interaction between two (or more) discussants.

Characteristic for the pragma-dialectical model is that in addition to the dialectical dimension we just highlighted, the model also takes the pragmatics of the discourse into account. This pragmatic dimension does justice to the fact that argumentation is inherently embedded in communicative interactions. In the ideal model, all of the discussion moves are therefore conceived of in terms of speech acts [5]. A Searlean speech act perspective on language use is integrated in the pragma-dialectical theory with a Gricean perspective on verbal interaction. This integration makes it possible to use pragmatic tools from discourse analysis to perform a dialectical reconstruction. Unexpressed or implicit parts of the discussion can be reconstructed on the basis of pragmatic insight by interpreting communicative acts as part of a discussion aimed at resolving a difference of opinion in a reasonable way.

The combination of a set of dialectical rules that prohibits the occurrence of fallacies and a pragmatic account of the communicative aspects of argumentation, makes the pragma-dialectical model a good starting point for the computational modelling of argumentative dialogue and the automated evaluation of argumentative discourse for fallacies. Because the ideal model of a critical discussion was not developed with such a computational application in mind, some preparatory steps are required. To explore the computational applications of the pragma-dialectical model, we propose to formalise it. This formalisation takes the form of a dialogue game for critical discussion (building on previous work [6]). By conceiving of the discussion model in terms of a game, the rules can be formulated in a fully disambiguated and explicit manner — i.e. as a formal dialogue system.

The critical discussion game is specified in terms of the Dialogue Game Description Language (or DGDL) [7]. The DGDL-specification has two main advantages. First, the standardised format facilitates a comparison to other dialogue systems that have been specified in terms of DGDL (such as Walton's CB). Second, the DGDL-specification makes the dialogue game computationally executable on the Dialogue Game Execution Platform (DGEP) [1].

Using DGEP makes it possible to play out a game of critical discussion through the online user interface Arvina ([arvina.arg.tech](http://arvina.arg.tech)) [3]. With Arvina, users can play out dialogues with others or with an artificial player modelled by the computer. During the game, the artificial player draws its dialogue contributions from the standpoints and arguments available in the AIFdb, an existing online repository [2]. Arvina offers a dialogical way of navigating the reasoning structures in the AIFdb, while at the same time adding the new arguments produced by the user to the existing structure. The critical discussion game ensures that fallacies are avoided while navigating and extending the AIFdb. Furthermore, making a playable game for critical discussion can shed light on the relation between the normative ideal model and the description of actual argumentation in the AIFdb.

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**Part III**

**QuestPro 2016**

## Invited talk: Questions as Information Types

Ivano Ciardelli  
Institute for Logic, Language and Computation  
University of Amsterdam  
sobekal@hotmail.com  
<http://www.ivanociardelli.altervista.org/>

Traditionally, logic is concerned with sentences of a particular kind, namely, statements. In this talk, I will argue that there is also an important role to be played in logic by questions. While statements can be seen as denoting specific pieces of information, questions can be seen as denoting information *types*: for instance, the question “where is Bob?” denotes the type of those pieces of information which specify Bob’s whereabouts. By generalizing the classical notion of entailment to questions, we can then capture in a unified way the classical relation of logical consequence, which holds between various pieces of information, as well as the relation of logical dependency, which holds between various information types. I will also discuss how questions may be used in inferences as placeholders for generic information of a given type; by manipulating such placeholders, we may construct formal proofs of the fact that certain dependencies hold. Interestingly, such proofs admit a kind of proofs-as-programs interpretation: they encode programs to transform information of the types denoted by the assumptions into information of the type denoted by the conclusion.

## Invited talk: Interacting and Reasoning across Domains

Jonathan Ginzburg  
UFR d'Études anglophones  
Université Paris-Diderot  
yonatan.ginzburg@univ-paris-diderot.fr  
<https://sites.google.com/site/jonathanginzburgswebsite/>

In this talk I will consider how theories of linguistically-based interaction can be extended to deal with interaction in other domains. I will start by considering how other-repair (e.g., clarification interaction) and metadiscursive-interaction (interaction concerning what to talk about) can be extended to self-repair. I will then consider how to integrate the import of certain non-verbal signals such as laughter or frowning into a model of dialogue interaction. Finally, I will consider the case of musical interaction. I will show how this widening of focus in theories of interaction can be achieved by incorporating into an erotetic approach the insights of enthymatic reasoning and of appraisal theories utilised in cognitive theories of emotion.

# Invited talk: A Game-Theoretic Analysis of Interrogative Games: First Steps

Yacin Hamami  
The Centre for Logic and Philosophy of Science  
Vrije Universiteit Brussel  
yacin.hamami@vub.ac.be

In the context of his so-called Interrogative Model of Inquiry, Hintikka has advocated an epistemological perspective according to which any information-seeking process shall be conceived as a questioning process with inherent strategic aspects, that is, as an interrogative game. Yet, no serious attempts have been made so far, either by Hintikka or others, to provide a full-fledged game-theoretic analysis of interrogative games. In this talk, I will explore the possibility of formalizing interrogative games using the resources of game theory, and to analyze thereby the strategic aspects of interrogative inquiry. To this end, I will propose two different, but complementary, game-theoretic formalizations of interrogative games. In the first one, interrogative games will be conceived as two-player extensive games with perfect information and chance moves, in which the two players will be referred to as 'Questioner' and 'Answerer', and in which the only chance move will occur at the very beginning of the game and will be used to determine the epistemic state of the Answerer. In this case, the probability distribution over the possible epistemic states of the Answerer will be an exogenous component of the game, making this formalization particularly suitable to represent interrogative games in the context of ordinary conversation. In the second one, interrogative games will be conceived as two-player extensive games with imperfect information, in which the two players will be referred to as 'Inquirer' and 'Nature', and in which Nature's choice of the actual world will be an entire part of Nature's strategic dimension. This formalization will therefore be particularly suitable to represent inquiry games as conceived by Hintikka in the Interrogative Model of Inquiry. In both cases, I will introduce the notions of questioning and answering strategies together with a formal apparatus to evaluate them, and thus to investigate the strategic aspects of interrogative inquiry. Furthermore, I will analyze the Nash equilibria of such interrogative games and will discuss their epistemological interpretation and significance. I will conclude the talk by relating the present analysis of interrogative games to the notion of erotetic search scenario introduced by Wiśniewski, and by pointing out to some possible applications of the resulting formal framework.

## Invited talk: Reasoning as Inquiry

Philipp Koralus  
University of Oxford  
philipp.koralus@stcatz.ox.ac.uk

I will make the case for the erotetic theory of reasoning, according to which we can make sense of both our natural tendency to make fallacious inferences and of our capacity for valid inference through the notion that reasoning is a form of inquiry.

# Some Operations on Erotetic Search Scenarios\*

Szymon Chlebowski  
Institute of Psychology  
Adam Mickiewicz University  
szymon.chlebowski@amu.edu.pl  
<http://sc52172.home.amu.edu.pl>

## Abstract

The notion of *erotetic search scenario* (*e-scenario*) is presented along with canonical operations on e-scenarios – embedding and contraction. New operations enabling elimination of different kinds of repetitions involved in e-scenarios are introduced.

**Keywords:** erotetic search scenarios, Inferential Erotetic Logic, logic of questions

## Extended abstract

An erotetic search scenario may be described as a labelled tree with a principal question at its root and possibly some declarative formulas (representing background knowledge; none of them can be an answer to the principal question) labelling consecutive nodes [3] (for a recent account see [1]). Then, each consecutive node is labelled by a formula that is either: (1) a declarative formula derived from the previous declarative formulas or a direct answer to the preceding auxiliary question; (2) a question which is erotetically implied by the previous questions (possibly on the basis of previous declarative formulas). Every node labelled by a declarative formula can be followed by at most one child-node. Each node labelled by a question has either one child node and it is labelled by a question, or several child nodes labelled by all the direct answers to the question which is a label of that node (in this case the question is called a query). Each leaf of such tree has to be labelled by a direct answer to the principal question. A properly built e-scenario has to contain at least one query.

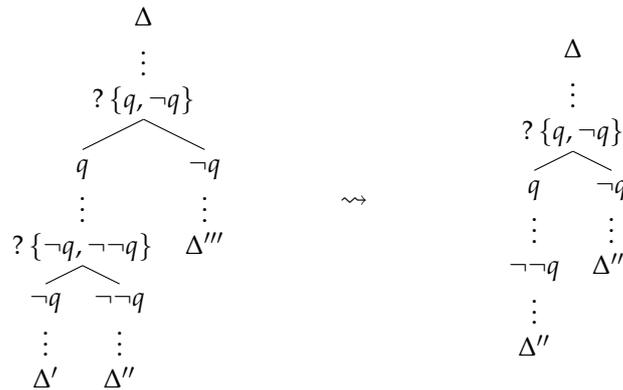
There are two canonical operations which may be performed on e-scenarios: *embedding* of one e-scenario into another and *contraction* of an e-scenario by a selected answer to a query. The embedding procedure was announced in [2] and defined in detail in [3]. Embedding can be performed when there exists an e-scenario  $\Sigma$  for the question which is a query of the e-scenario  $\Delta$ .  $\Sigma$  is embedded into  $\Delta$ , and e-scenario which is the result of embedding contains more specific instructions how to proceed in the process of solving problem expressed by the initial question. The operation of contraction has been introduced in [3]. Contraction of an e-scenario by the selected answer to a query amounts to removal of all the paths started by the query except the path containing that answer, and removing the considered query. Both operations have to

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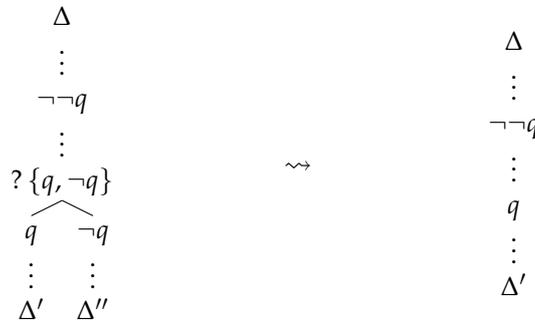
\*This work has been supported by the Polish National Science Center, grant no. 2012/04/A/HS1/00715.

conform to several restrictions and usually demand some syntactic transformations in order to produce e-scenarios.

The operation of embedding, when repeatedly applied to an e-scenario  $\Sigma$ , may cause some undesired side effects like co-occurrence of two equivalent queries on some path of  $\Sigma$ . On the left-hand side we have an e-scenario  $\Sigma$  with (at least two) equivalent queries. We assume that the first query is answered by  $q$ . The result of the application of one of the new operations is an e-scenario on the right-hand side. Query  $\{ \neg q, \neg\neg q \}$  is deleted by performing contraction on the answer consistent with the previous one.



There is also a possibility for an e-scenario to contain a path  $s$  such that somewhere on this path a declarative formula  $A$  occurs, and after this formula we have a query  $\{ B, \neg B \}$ , where  $B$  or  $\neg B$  is logically equivalent to  $A$ . It is not reasonable to request information about  $B$  when this information is already present. We will define an operation on e-scenarios which enables elimination of this kind of redundancy, like in this example:



We will define the notion of erotetic equivalence which holds between questions. We will introduce operations eliminating some kinds of redundancies of e-scenarios and we will show that these operations have desired properties.

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# *Abductive Question-Answer System for Non-Classical Logics\**

Szymon Chlebowski  
Andrzej Gajda  
Institute of Psychology  
Adam Mickiewicz University  
szymon.chlebowski@amu.edu.pl  
<http://sc52172.home.amu.edu.pl>

## **Abstract**

We propose an abductive procedure grounded in Inferential Erotetic Logic. The concept of an abductive rule is introduced and strategies of using these rules in the process of building up abductive hypothesis are discussed.

**Keywords:** abduction, modal logic, paraconsistent logic, inferential erotetic logic, erotetic calculi

## **1. Extended abstract**

The term ‘abduction’ was originally used to determine a kind of reasoning where new, hypothetical premise was added with the purpose of increasing the probability of the conclusion which was not acclaimed before (or was acclaimed with lesser firmness) [4, 5, p. 10]. From an algorithmic point of view the abductive hypothesis  $H$  “is legitimately dischargeable to the extent to which it makes it possible to prove (or compute) from a database a formula not provable (or computable) from it as it is currently structured” [2, p. 28]. We propose a framework named Abductive Question-Answer System (AQAS) which can be used to generate abductive hypotheses according to the above-mentioned approach.

In our presentation we focus on describing AQAS for classical propositional logic along with some non-classical logics such as paraconsistent logics normal modal logics. The whole structure is based on Wiśniewski’s Inferential Erotetic Logic (IEL) which enables us to transform an initial abductive question into auxiliary questions [7]. Answers to the auxiliary question are used to built up an answer we were seeking at the beginning, i.e. answer to the initial question. Through this process we obtain an abductive hypothesis. We also introduce abductive rules and strategies for using these rules for generating abductive hypotheses. The aim of this strategies is to guarantee that hypotheses generated are significant (the information which could not be

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proven or computed from the knowledge base is not a consequence only of our abductive hypothesis) and consistent with a given knowledge base. Introduced rules have questions as their premisses and propositions as their conclusions.

The following rules are examples of abductive rules. The intended meaning of the question  $?( \Gamma \vdash \Delta )$  which is the premiss of an abductive rule is the following: *what should be added to  $\Gamma$  to derive  $\Delta$ ?* The conclusion of this rule is an answer to the question-premiss.

$$\frac{?( \neg p, q, r \vdash q )}{p} \quad \frac{?( \neg p, q, r \vdash q )}{\neg r} \quad \frac{?( \neg p, q, r \vdash q )}{r \rightarrow q}$$

Note that in the above examples each conclusion is a correct answer to the respective question premiss. From  $\{ \neg p, q, r \} \cup \{ p \}$  one can derive  $q$ . Note also that not all correct answers generated by these rules are good: from  $\{ \neg p, q, r \} \cup \{ p \}$  one can derive  $q$ , but  $\{ \neg p, q, r \} \cup \{ p \}$  is inconsistent. Thus we design a strategy of using these rules by introducing certain restrictions.

The effect of applying such rules along with restrictions is that the set of possible hypotheses is reduced to the optimal one, i.e. redundant (non significant and inconsistent) cases are excluded when rules are applied with restrictions. As a result, Abductive Question-Answer System generates ‘good’ abductive hypotheses in one step, on the contrary to the more standard approach where this process is divided into two parts: generation of hypotheses and evaluation with qualifying selection (see for example [3]).

Our future work will cover also implementation of Abductive Question-Answer System in programming language. Part of this work is already finished — AQAS for classical propositional logic is implemented in Haskell programming language. This would enable us to test the system on huge datasets and compare it with already existing solutions, like one presented by Komosiński [3].

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# *Automated Generation and Analyzes of Erotetic Search Scenarios\**

Adam Kupś  
Institute of Psychology  
Adam Mickiewicz University  
adamku@amu.edu.pl

## **Abstract**

This work presents the results of a study concerning automatic generation and processing of Erotetic Search Scenarios (ESSs). The procedure for generation of ESSs has been implemented and used to obtain a set of ESSs. The ESSs have been evaluated using several criteria. The main part of this work focuses on the presentation of the results of multi-criteria dominance and discordance analyses on the evaluated ESSs. The final part discusses the results of the application of a rule mining algorithm to the obtained set of ESSs to gain a preliminary knowledge about the dependencies between different classes of ESSs.

**Keywords:** Erotetic Search Scenarios, optimization, rule mining

## **Extended abstract**

The presented work concerns automatic generation and analyses of Erotetic Search Scenarios. Erotetic Search Scenarios (ESSs) are formal constructs of a tree-like structure defined in Inferential Erotetic Logic developed by Andrzej Wiśniewski [5, 4]. ESSs enable finding an answer to a question, that may be based on some initial knowledge, by decomposing the initial and resulting questions into other questions. The decomposition of questions has to conform to the conditions of *erotetic implication*. This guarantees soundness of each step of the decomposition. Each path of an ESS represents a way to arrive at an answer (which is labeling a leaf) to the initial question. The considered ESS are expressed in the language of Classical Propostional Logic.

To generate ESSs the operation of embedding of one ESS into another has been employed. Embedding defined in [5] can be performed when the set of answers to the initial question of an embedded ESS is the same as the set of the answers to the question which is a query in the ESS into which the first ESS is embedded (let us call this first ESS a main scenario). A query is a question which introduces branching in the scenario, such that each answer to the query labels the first node of exactly one of the branches. Informally speaking, the operation of embedding

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places the embedded scenario at the place of a query in the main scenario. The operation is admissible if both of the scenarios satisfy Embedding Theorem.

The presented approach, apart from the embedding operation, makes use of the contraction operation which is a removal of paths of the generated scenarios by “answering” a selected query. This operation is performed by replacing a query with one of the answers to this query and by removal of all of the branches occurring immediately after this query, apart from the one starting with the selected answer. To eliminate redundancies and inconsistencies that can occur after the embedding, the contraction operation alongside some more support operations, that remove repeating occurrences of formulas, are performed.

The procedure of the automatic generation of ESSs made use of the initial set of fifteen manually created scenarios. This set was subjected to a few steps of embedding of one scenario into another (when possible) and “repairing” of the resulting scenarios by the contraction and support operations. Additionally, each step of embedding and repair was followed by generation of canonical and concise forms of the previously obtained ESS. After the generation procedure, all the resulting ESS have been evaluated on seven criteria: completeness, conciseness, canonicity, purity, redundancy, realizability and overflow.

After the generation and evaluation stages, three kinds of analyses have been performed. First, all the obtained scenarios have been analyzed using multi-criteria dominance relation (see [3] for an application of multi-criteria dominance analysis in computational logic). The multi-criteria dominance relation allows comparison of the scenarios evaluated with many criteria without imposing any order of the preference on those criteria. The result of this kind of analysis allows to establish a group of non-dominated ESSs, which can be considered the best or the most efficient ones. Next, the data about the criteria have been used to perform multi-criteria discordance analysis. Multi-criteria discordance analysis helps to gain knowledge about interdependencies between the criteria to better understand to what degree these criteria are independent from each other and how they actually work in the generated set of ESSs. Finally, Apriori rule mining algorithm [1, 2] has been employed to get even more knowledge about the dependencies between the criteria in the set of ESSs. The output of this algorithm consists of conditional sentences that represent rules combining information about values of different criteria. Moreover, the degree of confidence of those rules is provided. This is why the algorithm has been also used to search for general statements about ESSs by analyzing the rules of maximal confidence.

The results shed a new light on the concept of optimal ESS and indicate how to improve the procedures to generate efficient scenarios. The results of application of Apriori algorithm allow to establish general statements about relations between different classes of Erotetic Search Scenarios. The future plans concern: research of complexity issues related to generation and application of ESSs, extension of the used language to the first-order language, inclusion of more evaluation (also quantitative) criteria and automation of the analysis of rules generated by Apriori algorithm.

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# *Dialogic logic and dynamisation of erotetic search scenarios execution\**

Paweł Łupkowski  
Institute of Psychology  
Adam Mickiewicz University  
Pawel.Lupkowski@amu.edu.pl  
<http://reasoning.edu.pl>

## **Abstract**

In this talk I will present a dialogic logic approach to the dynamisation of erotetic search scenario (ESS) execution. ESS is a tool from Inferential Erotetic Logic. The outcome system allows for the modelling of the dynamic interrogative problem solving with the underlying IEL concepts of erotetic inferences validity for a simple case of two party dialogue game and for more complex multi-agent environment. I will also discuss applicability of this approach into the field of cooperative answering for databases and information systems.

**Keywords:** Inferential Erotetic Logic (IEL), erotetic search scenario, questions, dialogue games

## **1. Motivations**

In the talk I will focus on a particular kind of information seeking dialogues [9], namely the ones where a problem is solved *via* the questioning process. An agent solves the problem by dividing it into several simpler sub-problems and by gathering solutions to these sub-problems from other agents, thus obtaining a solution to the initial problem. Such a process may be successfully modelled using erotetic search scenario (a tool developed within the Inferential Erotetic Logic, IEL, [13], [14]). ESS represents a certain map of possible courses of events in a given questioning process. Each path of an e-scenario is one of the ways the process might go depending on the answers obtained to queries.

I propose a system that combines IEL approach with a dialogue logic. For this purpose we will consider a dialogue as a certain game, which consists of a sequence of locution events [3]. As a result we obtain a modular system (which is easy to modify and adapt) that allows us to describe a game where one of the players is using e-scenarios as a questioning strategy.

The motivation for such an approach is twofold. Firstly, it comes from the system presented by [10], where a dynamic epistemic component is added to a logic of questions in order to describe an information seeking procedure. Secondly, my approach is inspired by the system pro-

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posed in [1] and [5]. My approach shares the main intuition of using a logic in the background of a formal dialogue system in order to check the correctness of certain dialogue moves.

## 2. Dialogue games

Let us imagine a dialogue between two participants: Bob and Alice. This dialogue is a questioning game in which Bob wants to solve a certain problem expressed by his initial question. He decomposes the initial question into series of auxiliary questions, that will be asked of Alice. By obtaining answers to auxiliary questions Bob hopes to solve the initial problem. We assume that he will use an ESS as his questioning strategy. We also assume that Alice is providing information according to the best of her knowledge. She may, however, withdraw from providing an answer.

Intuitively we may describe the game as follows. Bob asks questions and Alice provides answers. The game goes step-by-step. In one step Bob is allowed to: ask a question or provide a justification for a question/solution; or end the game. Alice in one step can: provide an answer to a question; challenge the question (“I claim that your question is irrelevant”, which in our case is understood as not fulfilling the conditions of e-implication); challenge the solution (“Please explain the solution you propose”); deny answering (“I don’t want to answer”).

In a scenario where one of his questions is being attacked Bob is obliged to provide erotetic justification for the question (i.e. reveal a part of his questioning strategy).

I propose a dialogue logic  $DL(IEL)_2$  in order to precisely describe the rules of the game and the rules of executing Bob’s questioning strategy. In the talk I will specify: (i) The taxonomy of locutions; (ii) Commitment store rules; and (iii) Interaction rules for  $DL(IEL)_2$ . I will also present how to extend this system in order to grasp the multi-agent interaction (where Bob employs more than one information source in his questioning process).

## 3. Cooperative answering

Databases and information systems in general offer correct answers to user’s questions (as far as these systems contain valid data). The problem is to ensure that the answers will also be non-misleading and useful for a user—i.e. *LupQ:cooperative* [2]. Certain cooperative answering phenomena may be modelled within the framework of ESS [8], [6]. ESS are used here to generate cooperative responses [7] in cases when: the answer to a question is negative or there is no answer available in a database. These techniques are supplemented with the idea of cooperative question-responses as an extension of traditional cooperative behaviours.

I propose a dialogue logic systems  $DL(IEL)_{COOP1}$  and  $DL(IEL)_{COOP2}$  as an integrative approach for cooperative techniques based on ESS. The first system is able to generate cooperative responses without the cooperative questioning capacity.  $DL(IEL)_{COOP2}$  is extended in such a way that it is able to interact with a user using cooperative questioning. Dialogue logic systems allows us to present, in a clear way, interactions between cooperative information system and a user in a step-by-step dialogue manner as they are governed by the interaction rules.

## 4. Summary

The proposed framework is fairly universal and modular. It can be modified on the level of rules used to describe the dynamics of questioning process. E.g. it allows for adding new

locution types, or modifying interaction rules to obtain the desired behaviours of agents in interaction. What is more we can also adapt IEL tools used here in order to tailor them better to our needs (one can e.g. use the ESS based on the notion of weak e-implication [11], falsificationist e-implication [4] or epistemic e-implication [10]).

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# *Erotetic Epistemic Logic*<sup>\*</sup>

Michal Peliš  
Department of Logic  
Faculty of Arts  
Charles University in Prague  
michal.pelis@ff.cuni.cz

## Abstract

Our aim is to present a possible erotetic enrichment of epistemic logic. Here we omit the dynamic aspect of communication with questions. Erotetic items that will be introduced in the epistemic framework are: formalization of questions in modal language, extension of satisfaction relation for questions, answerhood conditions, and inferential relations with questions. A specialist in erotetic logic will recognize the significant influence of Inferential Erotetic Logic (IEL) [4, 5] and the intensional approach from [1, 2]. Both approaches inspired questions' formalization and semantics of questions. IEL inspired various inferential structures with questions. This paper is based on the third chapter of [3].

**Keywords:** epistemic logic, erotetic logic, askability

## 1. Epistemic logic with questions

Logic in use will be the normal multi-modal logic S5 with its standard relational semantics (Kripke frames and models). The language has signs for atomic formulas  $\mathcal{P} = \{p, q, \dots\}$  and formulas defined in BNF as follows:  $\varphi ::= p \mid \neg\psi \mid \psi_1 \wedge \psi_2 \mid K_i\psi \mid \hat{K}_i\psi$ .<sup>1</sup> In multi-agent variants of epistemic logic we presuppose that there is a finite set of agents  $\mathcal{A}$ .

A *Kripke frame* is a relational structure  $\mathcal{F} = \langle S, R_i \rangle$  with a set of states (possible worlds)  $S$ , and an accessibility relation (equivalence)  $R_i \subseteq S^2$  for (every)  $i \in \mathcal{A}$ . A *Kripke model*  $\mathbf{M}$  is a pair  $\langle \mathcal{F}, v \rangle$  where  $v$  is a valuation of atomic formulas. The satisfaction relation  $\models$  between states and formulas is defined in the standard way.

We extend the epistemic language with brackets  $\{, \}$  and the question mark  $?_x$ , where  $x$  indicates the questioner(s), it can be a 'name' of an agent or a group of agents. A question  $Q^x$  is any formula of the form  $?_x\{\psi_1, \dots, \psi_n\}$ , where  $dQ^x = \{\psi_1, \dots, \psi_n\}$  is the set of direct answers to a question  $Q$ . Direct answers are formulas of our extended epistemic language (questions can

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<sup>1</sup>The necessity-like modality  $K_i$  can be interpreted as 'agent  $i$  knows that ...'. Modality  $\hat{K}_i$  is understood to be dual to  $K_i$ :  $\hat{K}_i\varphi \equiv \neg K_i\neg\varphi$ .

<sup>\*</sup>The work was supported by grant From Shared Evidence to Group Attitudes (Czech Science Foundation, no. 16-07954J).

be among direct answers). We suppose that  $dQ^x$  is finite with at least two syntactically distinct elements.

It makes little sense to speak about the truth or falsity of a question. We introduce instead a notion of *askability* of a question.

**Definition 3** (Individual askability). *We say that  $Q^i = ?_i\{\alpha_1, \dots, \alpha_n\}$  is askable by an agent  $i$  in the state  $(M, w)$  and write  $(M, w) \models Q^i$  iff*

1.  $(M, w) \not\models K_i\alpha$ , for each  $\alpha \in dQ^i$  (non-triviality)
2.  $(M, w) \models \hat{K}_i\alpha$ , for each  $\alpha \in dQ^i$  (admissibility)
3.  $(M, w) \models K_i\left(\bigvee_{\alpha \in dQ^i} \alpha\right)$  (context)

Moreover, we can speak about *group askability* whenever the question is askable for each agent from a (non-empty) group  $G \subseteq \mathcal{A}$ .

We will discuss all three conditions of askability: *Non-triviality* and *admissibility* are connected with answerhood conditions of questions, and *context* plays a special role connected with presuppositions of questions. Simultaneously, we will clarify the role of questions as direct answers and some problematic aspects of the presented approach.

## 2. Epistemic erotetic implication

Erotetic inference is implicitly based on (standard) implication. We say that a question  $Q_1$  implies  $Q_2$  (in a state  $s$ , for an agent  $i$ ) whenever askability of  $Q_1$  (in  $s$ , for  $i$ ) implies askability of  $Q_2$  (in  $s$ , for  $i$ ).

$$(M, w) \models Q_1^i \rightarrow Q_2^i \text{ iff } (M, w) \models Q_1^i \text{ implies } (M, w) \models Q_2^i$$

We can say that  $Q_1$  *entails*  $Q_2$  if and only if the formula  $Q_1^i \rightarrow Q_2^i$  is valid (for any agent  $i$ ).

In the talk, we will discuss the role of epistemic erotetic implication in answerhood conditions.

## 3. Other inferential structures with questions

A question  $Q$  is askable (by an agent  $i$ ) in  $(M, w)$  with respect to a set of formulas  $\Gamma$  iff  $(M, w) \models \{K_i\gamma \mid \gamma \in \Gamma\}$  and  $(M, w) \models Q^i$ . We write:

$$(M, w) \models (K_i\Gamma \bowtie Q_1^i)$$

If, moreover,  $Q_2^i$  is implied (by  $Q_1^i$  with respect to  $\Gamma$ ) in this state, we write

$$(M, w) \models (K_i\Gamma \bowtie Q_1^i) \Rightarrow Q_2^i$$

A question  $Q$  is askable (by an agent  $i$ ) in  $(M, w)$  with respect to a set of hypotheses  $\Gamma$  iff  $(M, w) \models \{K_i\gamma \mid \gamma \in \Gamma\}$  implies  $(M, w) \models Q^i$ . We write:

$$(M, w) \models K_i\Gamma \Rightarrow Q^i$$

Properties of these inferential structures will be presented in the talk.

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# *On non accepting Hamblin's third postulate\**

Mariusz Urbański  
Institute of Psychology  
Adam Mickiewicz University  
mariusz.urbanski@amu.edu.pl  
<http://mu.edu.pl>

## **Abstract**

In this paper I argue that there are sound reasons not to accept Hamblin's postulate that the possible answers to a question are an exhaustive set of mutually exclusive possibilities.

**Keywords:** Logic of questions, Hamblin's postulates, safe questions

## **Extended abstract**

In his paper "Questions" Hamblin [3, p. 162–164] famously formulated three postulates which were to characterize questions:

1. Knowing what counts as an answer is equivalent to knowing the question.
2. An answer to a question is a statement.
3. The possible answers to a question are an exhaustive set of mutually exclusive possibilities.

None of these postulates is uncontroversial and each can be argued for and against (see for example [1, 2, 4]). The focus in this paper is on the third one: my point is that there are sound reasons for this postulate to be rejected or at least to be considered with greater care than it is usually the case.

There are two conditions imposed by the third postulate on a set of possible answers to a question: the exhaustiveness condition and the mutual exclusiveness condition. Hamblin justifies the exhaustiveness condition with an example of a question "Have you stopped beating your wife?". It is seemingly safe question (i.e. it has true direct answer [7, p. 38]), but in fact it may happen that no direct answer to it is true, if the presupposition of the question is false. Hamblin calls this question "logically improper", because, in his own words, "the indicated answers 'yes' and 'no' do not, on the usual reckoning, cover all the logical possibilities" [3, p.

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163]. To make this question logically proper one needs to add a ‘supplementary’ answer, invalidating the presupposition, e. g. “I have never beaten my wife” (nb. Hamblin himself indicated that the safeness of a question can be considered not in an unrestricted sense but relatively to a certain presupposition or assumption). As for the mutual exclusiveness condition, Hamblin’s justification of it can be construed in terms of Gricean maxim of quantity (and also of manner, to some extent): “Either Europe, or Asia, or Africa” is not a proper answer to the question “In which continent is Luxembourg”, as it does not convey the desired amount of information, or, in Hamblin’s words, is not ‘complete’ [3, p. 164].

Thus it is rather obvious that the rationale underlying the third postulate has a strong pragmatic flavour to it. In particular, by no means it amounts to its received interpretation: that a disjunction of all the direct answers to the question should be a valid formula. We can interpret the third postulate as a claim that there should be a possibility for each question to be truthfully answered, even if its presuppositions are false.

My claims in this paper are the following:

1. The third postulate taken seriously has important consequences first of all for semantics of questions: classical logic as the basis for it becomes untenable. Also, the formal representations of questions become affected (in particular the ones following the principles of the set-of-answers methodology [4]).
2. There are questions which obey the third postulate (elsewhere [5] we called them Maximally Informative Questions); nevertheless, not all the questions do.
3. There are good reasons to reject both exhaustiveness and mutual exclusiveness conditions, based on the real linguistic practice.
4. What Hamblin puts into one category of possible answers should be divided into direct answers, non-direct yet admissible answers and, possibly, other responses to a question.

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# *Deduction and reduction theorems for Inferential Erotetic Logic*

Andrzej Wiśniewski  
Institute of Psychology  
Adam Mickiewicz University  
Andrzej.Wisniewski@amu.edu.pl  
<http://andrzejwisniewski.edu.pl/>  
<https://intquestpro.wordpress.com>

## **Abstract**

A new method of defining the class of admissible partitions of a formal language enriched with questions will be presented. Admissible partitions defined according to the method are basically proof-theoretic entities. Then some new results which seem to deserve the label deduction theorems for question evocation and erotetic implication will be discussed. Finally, I will show how question evocation by a non-empty set of d-wffs can be reduced to question evocation by the empty set, and how erotetic implication based on a non-empty set of d-wffs can be reduced to pure erotetic implication.